ASSIGNMENT 2
MTH102A

(1) Using Gauss Jordan elimination method find the inverse of $\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$.

(2) Let $\sigma \in S_5$ be given by

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 4 & 1 & 2 & 3 \end{pmatrix}$$

(a) Find sign of σ and sign of σ^{-1},
(b) Find $\sigma^2 = \sigma \circ \sigma$.

(3) Using the definition compute the determinant of $\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$.

(4) Let A be a square matrix of order n. Show that $det(A) = 0$ if and only if there exists a non-zero vector $X = (x_1, x_2, ..., x_n)$ such that $AX^T = 0$.

(5) (Vandermonde Matrix) Find the determinant of the following matrix:

$$\begin{pmatrix} 1 & x_1 & x_1^2 & ... & x_1^{n-1} \\ 1 & x_2 & x_2^2 & ... & x_2^{n-1} \\ .. & .. & .. & ... & .. \\ 1 & x_n & x_n^2 & ... & x_n^{n-1} \end{pmatrix}$$

(6) Let A be a $n \times n$ real matrix. Show that $det(adj(A)) = (det(A))^{n-1}$ and $adj(adj(A)) = (det(A))^{n-2}.A$.

(7) Using Cramer's rule solve the following system:

$$x + 2y + 3z = 1$$
$$-x + 2z = 2$$
$$-2y + z = -2$$