ASSIGNMENT 1
MTH102A

(1) Show that matrix multiplication is associative i.e. $A(BC) = (AB)C$ whenever the multiplication is defined.

(2) Suppose A and B are matrices of order $m \times n$ such that $A\bar{x} = B\bar{x}$ for all $\bar{x} \in \mathbb{R}^n$. Prove that $A = B$.

(3) Let $A = (a_{ij})$ be a matrix. Transpose of A, denoted by A^T, is defined to be $A^T = (b_{ij})$ where $b_{ij} = a_{ji}$.
(i) Show that $(A + B)^T = A^T + B^T$, whenever $A + B$ is defined
(ii) Show that $(AB)^T = B^T A^T$, whenever AB is defined.

(4) A square matrix A is said to be *symmetric* if $A = A^T$ and a square matrix A is said to be *skew symmetric* if $A = -A^T$.
Prove that a square matrix can be written as a sum of symmetric and a skew symmetric matrix.

(5) A square matrix A is said to be *nilpotent* if $A^n = 0$ for some natural number n.
(i) Give examples of non-zero nilpotent matrices,
(ii) Prove that if A is nilpotent then $A + I$ is an invertible matrix, where I is the identity matrix.

(6) Trace of a square matrix A, denoted by $Tr(A)$, is defined to be the sum of all diagonal entries.
(i) Suppose A, B are two square matrices of same order. Prove that $Tr(AB) = Tr(BA)$.
(ii) Show that for an invertible matrix A, $Tr(ABA^{-1}) = Tr(B)$.

(7) Apply Gauss elimination method to solve the system $2x+y+2z = 3$, $3x-y+4z = 7$ and $4x + 3y + 6z = 5$.

(8) Find the row reduced echelon form of the matrix $\begin{bmatrix} 2 & 6 & -2 \\ 3 & -2 & 8 \end{bmatrix}$.