ASSIGNMENT 3
MTH102A

(1) In $\mathbb{R}$, consider the addition $x \oplus y = x + y - 1$ and the scalar multiplication $\lambda.x = \lambda(x-1)+1$. Prove that $\mathbb{R}$ is a vector space over $\mathbb{R}$ with respect to these operations. What is the additive identity (the **0** vector in the definition) in this case ?

(2) Show that $W = \{(x_1, x_2, x_3, x_4) : x_4 - x_3 = x_2 - x_1\}$ is a subspace of $\mathbb{R}^4$ spanned by vectors $(1, 0, 0, -1), (0, 1, 0, 1), (0, 0, 1, 1)$.

(3) Describe all the subspaces of $\mathbb{R}^3$.

(4) Find the condition on real numbers a, b, c, d so that the set $\{(x, y, z)|ax + by + cz = d\}$ is a subspace of $\mathbb{R}^3$.

(5) Discuss the linear dependence/independence of following set of vectors:
(i) $\{(1, 0, 0), (1, 1, 0), (1, 1, 1)\}$ in $\mathbb{R}^3$ as a vector space over $\mathbb{R}$,
(ii) $\{(1, 0, 0, 0), (1, 1, 0, 0), (1, 1, 1, 0), (3, 2, 1, 0)\}$ in $\mathbb{R}^4$ as a vector space over $\mathbb{R}$,
(iii) $\{(1, i, 0), (1, 0, 1), (i + 2, -1, 2)\}$, in $\mathbb{C}^3$ as a vector space over $\mathbb{C}$,
(iv) $\{(1, i, 0), (1, 0, 1), (i + 2, -1, 2)\}$, in $\mathbb{C}^3$ as a vector space over $\mathbb{R}$,
(v) The sets $\{1, sinx, cosx\}$ and $\{2, sin^2x, cos^2x\}$ in the vector space of real valued functions $F = \{f : f : \mathbb{R} \to \mathbb{R}\}$.
(v) $\{u + v, v + w, w + u\}$ in a vector space V given that $\{u, v, w\}$ is linearly independent.

(6) Let $W_1 = Span\{(1, 1, 0), (-1, 1, 0)\}$ and $W_2 = Span\{(1, 0, 2), (-1, 0, 4)\}$. Prove that $W_1 + W_2 = \mathbb{R}^3$.

(7) Find 3 vectors u, v and w in $\mathbb{R}^4$ such that $\{u, v, w\}$ is linearly dependent whereas $\{u, v\}, \{u, w\}$ and $\{v, w\}$ are linearly independent. Extend each of the linearly independent sets to a basis of $\mathbb{R}^4$.

(8) Let A be a $n \times n$ matrix over $\mathbb{R}$. Then A is invertible iff the row vectors are linearly independent over $\mathbb{R}$ iff the column vectors are linearly independent over $\mathbb{R}$.

(9) Determine if the set $T = \{1, x^2 - x + 5, 4x^3 - x^2 + 5x, 3x + 2\}$ is a basis for the vector space of polynomials in x of degree ≤ 4. Is this set a basis for the vector space of polynomials in x of degree ≤ 3 ?