ASSIGNMENT 4
MTH102A

(1) Let $\{w_1, w_2, ..., w_n\}$ be a basis of a finite dimensional vector space V. Let v be a non zero vector in V. Show that there exists w_i such that if we replace w_i by v in the basis it still remains a basis of V.

(2) Find the dimension of the following vector spaces :
(i) X is the set of all real upper triangular matrices,
(ii) Y is the set of all real symmetric matrices,
(iii) Z is the set of all real skew symmetric matrices,
(iv) W is the set of all real matrices with $Tr(A) = 0$

(3) Let $\mathcal{P}_2(X, \mathbb{R})$ be the vector space of all polynomials in X of degree less or equal to 2. Show that $B = \{X + 1, X^2 - X + 1, X^2 + X - 1\}$ is a basis of $\mathcal{P}_2(X, \mathbb{R})$. Determine the coordinates of the vectors $2X - 1, 1 + X^2, X^2 + 5X - 1$ with respect to B.

(4) Let W be a subspace of a finite dimensional vector space V
(i) Show that there is a subspace U of V such that $V = W + U$ and $W \cap U = \{0\}$,
(ii) Show that there is no subspace U of V such that $W \cap U = \{0\}$ and $dim(W) + dim(U) > dim(V)$.

(5) Decide which of the following maps are linear transformations:
(i) $T : \mathbb{R}^3 \to \mathbb{R}^3$ defined by $T(x, y, z) = (x + 2y, z, |x|)$,
(ii) Let $M_n(\mathbb{R})$ be the set of all $n \times n$ real matrices and $T : M_n(\mathbb{R}) \to M_n(\mathbb{R})$ defined by
(a) $T(A) = A^T$,
(b) $T(A) = I + A$, where I is the identity matrix of order n,
(c) $T(A) = BAB^{-1}$, where $B \in M_n(\mathbb{R})$ is an invertible matrix.

(6) Let $T : \mathbb{C} \to \mathbb{C}$ be defined by $T(z) = \overline{z}$. Show that T is $\mathbb{R}$-linear but not $\mathbb{C}$-linear.

(7) Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation such that $T(1, 0, 0) = (1, 0, 0), T(1, 1, 0) = (1, 1, 1), T(1, 1, 1) = (1, 1, 0)$. Find $T(x, y, z)$, $Ker(T)$, $R(T)$ (Range of T). Prove that $T^3 = T$.

(8) Find all linear transformations from $\mathbb{R}^n$ to $\mathbb{R}$.

(9) Let $\mathcal{P}_n(X, \mathbb{R})$ be the vector space of all polynomials in X of degree less or equal to n. Let T be the differentiation transformation from $\mathcal{P}_n(X, \mathbb{R})$ to $\mathcal{P}_n(X, \mathbb{R})$. Find $Range(T)$ and $Ker(T)$.