ASSIGNMENT 6
MTH102A

(1) Let A and B be square matrices of same order. Prove that characteristic polynomials of AB and BA are same. Do AB and BA have same minimal polynomial ?

(2) Let A be an $n \times n$ matrix. Show that A and A^T have same eigen values. Do they have the same eigen vectors ?

(3) Find the characteristic and minimal polynomial of the following matrix and decide if this matrix is diagonalizable.

$$A = \begin{bmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{bmatrix}$$

(4) Find the inverse of the matrix $\begin{pmatrix} -1 & 2 & 0 \\ 1 & 1 & 0 \\ 2 & -1 & 2 \end{pmatrix}$ using the Cayley-Hamilton theorem.

(5) Diagonalize $A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & -3 \end{bmatrix}$ and compute A^{2019}.

(6) Let W be the subspace of $\mathbb{R}^4$ spanned by $\{u_1 = (1,1,1,1), u_2 = (2,4,1,5), u_3 = (2,0,4,0)\}$. Using the standard Euclidean inner product on $\mathbb{R}^4$ find an orthogonal basis for W.

(7) Consider $P_2(\mathbb{R})$ together with inner product: $\langle p(x), q(x) \rangle = \int_0^1 p(x)q(x)\, dx$. Find an orthogonal basis for $P_2(\mathbb{R})$.

(8) Is the following matrix orthogonally diagonalizable ? If yes, then find P such that PAP^T is diagonal.

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

(9) Find a singular value decomposition of the matrix $A = \begin{pmatrix} -2 & 2 \\ -1 & 1 \\ 2 & -2 \end{pmatrix}$.

(10) Let $M_{n\times n}$ be the vector space of all real $n \times n$ matrices. Show that $\langle A, B \rangle = Tr(A^T B)$ is an inner product on $M_{n\times n}$. Show that the orthogonal complement of the subspace of symmetric matrices is the subspace of skew-symmetric matrices, i.e., $\{A \in M_{n\times n} \mid A \text{ is symmetric}\}^\perp = \{A \in M_{n\times n} \mid A \text{ is skew-symmetric}\}$.