MTH-102

Linear Algebra (LA)

Ordinary differential Equation (ODE)

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Quiz-1 - 20

Midsem - 60

Quiz-2 - 20

Endsem - 100

$$\mathbb{R} = \mathbb{Q} \cup \mathbb{Q}'$$

a

$$\mathbb{N} = \{1, 2, 3, \ldots\}$$

$$\mathbb{C} = \{a + ib : a, b \in \mathbb{R}\}$$

$$\mathbb{Z} = \{\ldots -4, -3, -2, -1, 0, 1, 2, \ldots\}$$

$$\mathbb{Q} = \left\{\frac{p}{q} : p, q \in \mathbb{Z},\ q \neq 0\right\}$$

$a \in N$

Z

$x^2+1=0$

$C = \{a+ib : a, b \in R\}$

$$A = (a_{ij})_{m \times n} \quad \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & & a_{2n} \\ & & & \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}$$

a_{ij}

$A \quad B$

$m \times n$

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} + B = \begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ \vdots & & & \\ b_{m1} & & \cdots & b_{mn} \end{pmatrix}$$

$$C = (c_{ij})$$

$$c_{ij} = a_{ij} + b_{ij}$$

$$A_{2\times 3} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{pmatrix}_{2\times 3}$$

$$C = \begin{pmatrix} a_{11}b_{11} & a_{12}b_{12} & a_{13}b_{13} \\ \cdot & - & - \end{pmatrix}$$

$$A_{m \times n} \quad B_{m \times n}$$

$$C = \begin{pmatrix} a_{11}B & a_{12}B & & a_{1n}B \\ & & & \\ a_{m1}B & \cdots & & a_{mn}B \end{pmatrix}$$

$$\alpha A = \begin{pmatrix} \alpha a_{11} & \alpha a_{12} & \cdots \\ & & \end{pmatrix}$$

$$A_{m\times n} = (a_{ij}) \qquad B_{n\times k} = (b_{ij})$$

$$C = (c_{ij})_{n}$$

$$c_{ij} = \sum_{k=1}^{n} a_{ik} b_{kj}$$

$$A + B = 0$$

$$c_{11} = \sum_{k=1}^{n} a_{1k} b_{k1}$$

$$= a_{11} b_{11} + a_{12} b_{21} + \cdots$$

$$=$$

$$A + B = B + A$$

$$A + (B + C) = (A + B) + C$$

$$(AB)C = A(BC)$$

$$AB = BA \,?$$

$$\underset{A}{\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}} \underset{B}{\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}}$$

$$A(B+C)$$
$$= AB + AC$$

$$B = (A^T)^T = A$$

$$A_{m\times n}$$

$$B = (b_{ij})$$

$$b_{ij} = a_{ji}$$

$$A_{m\times m} \quad \begin{pmatrix} & x & x & \\ 0 & & x & x \\ 0 & & & x \end{pmatrix}$$

– upper triangular

Symmetric if $A = A^T$
Skew symmetric if $A = -A^T$

$$\begin{pmatrix} 1 & & 0 \\ & 1 & & 0 \\ 0 & & 1 & \\ & 0 & & 1 \end{pmatrix}_{n \times n}$$

identity matrix

$$ax + by = c$$

$$a, b, c \in \mathbb{Z}$$

$$a_1 x + b_1 y + c_1 z = d_1$$

$$a_2 x + b_2 y + c_2 z = d_2$$

$$a_1 x + b_1 y = c_1$$

$$a_2 x + b_2 y = c_2$$

$$x + y = 3 \quad -2$$

$$2x + y =$$

$$x + y = 3$$

$$2x + 3y = 4.$$

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = d_1$$

$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = d_2$$

$$\vdots$$

$$a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n = d_m$$

$$A\underline{X} = \underline{d}$$

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ \vdots & & & \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}$$

$$\underline{X} = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}$$

$$\underline{d} = \begin{pmatrix} d_1 \\ d_2 \\ \vdots \\ \end{pmatrix}$$

(1) $A x = \underline{d}$

(2) $B x = \underline{c}$

(1) and (2) are equivalent if every equation in (1) is a linear combination of equations of (2) and viceversa.