$$E_p \cdots E_2 E_1 A F_1 F_2 \cdots F_q = \left(\begin{array}{c|c} I_{r\times r} & 0 \\ \hline 0 & 0 \end{array}\right)$$

A [illegible]

$$E_p \cdots E_1 A = I$$

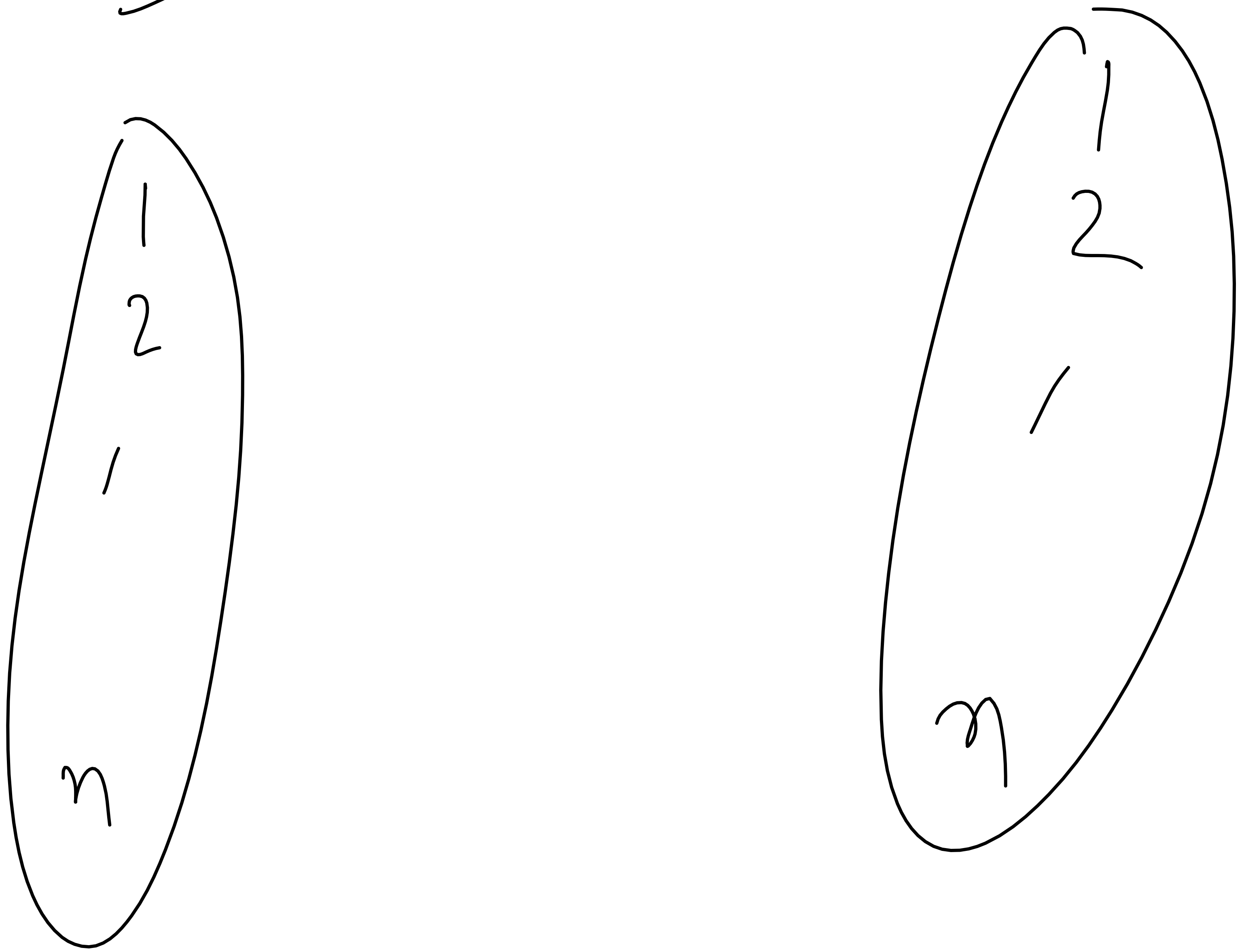
$$A^{-1} = E_p \cdots E_1 I$$

$$E_p \cdots E_2 E_1 \left(\begin{array}{c|c} A & I \end{array}\right)$$

$$\left(\begin{array}{c|c} I & A^{-1} \end{array}\right)$$

$S = \{1, 2, 3, \ldots n\}$

$f : S \longrightarrow S$



S_n = Set of all permutation on S

= Set of all bijection from $S \to S$

$S \xrightarrow{f} S \xrightarrow{g} S$

$g \circ f \in S_n$

$f \circ g \in S_n$

$f^{-1} \in S_n$

$id \in S_n$

S_n $\qquad$ $\sigma, \tau \in S_n$

$\sigma \circ \tau \in S_n$ $\qquad$ $\tau \circ \sigma \in S_n$

$\sigma \in S_n$

$S_n =$ Set of all permutation

$\sigma \circ S_n = \{ \sigma \circ \tau : \tau \in S_n \}$

$S_n = \sigma \circ S_n$

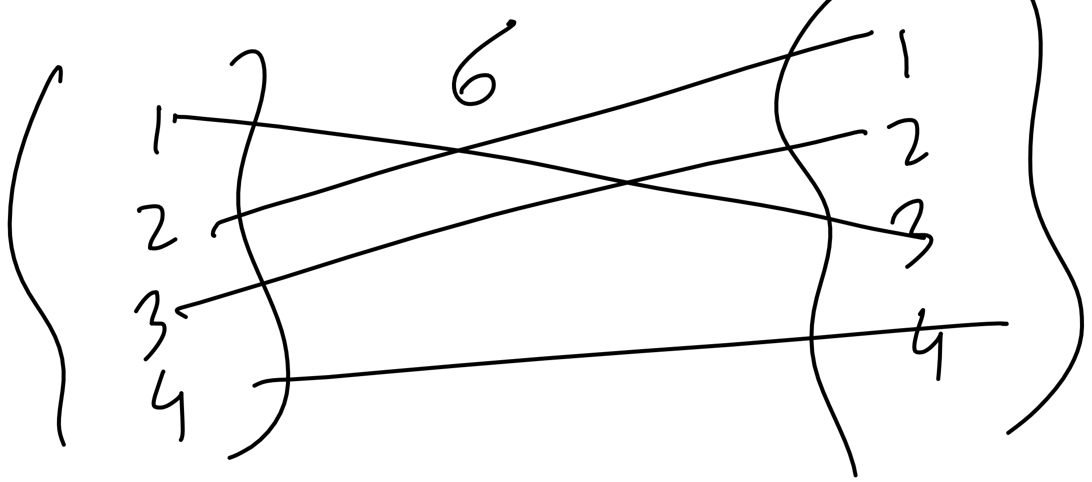
$S_n \circ \sigma = S_n$

$\sigma \circ \tau_1 = \sigma \circ \tau_2$

$\sigma^{-1} \cdot \sigma \circ \tau_1 = \sigma^{-1} \sigma \cdot \tau_2$

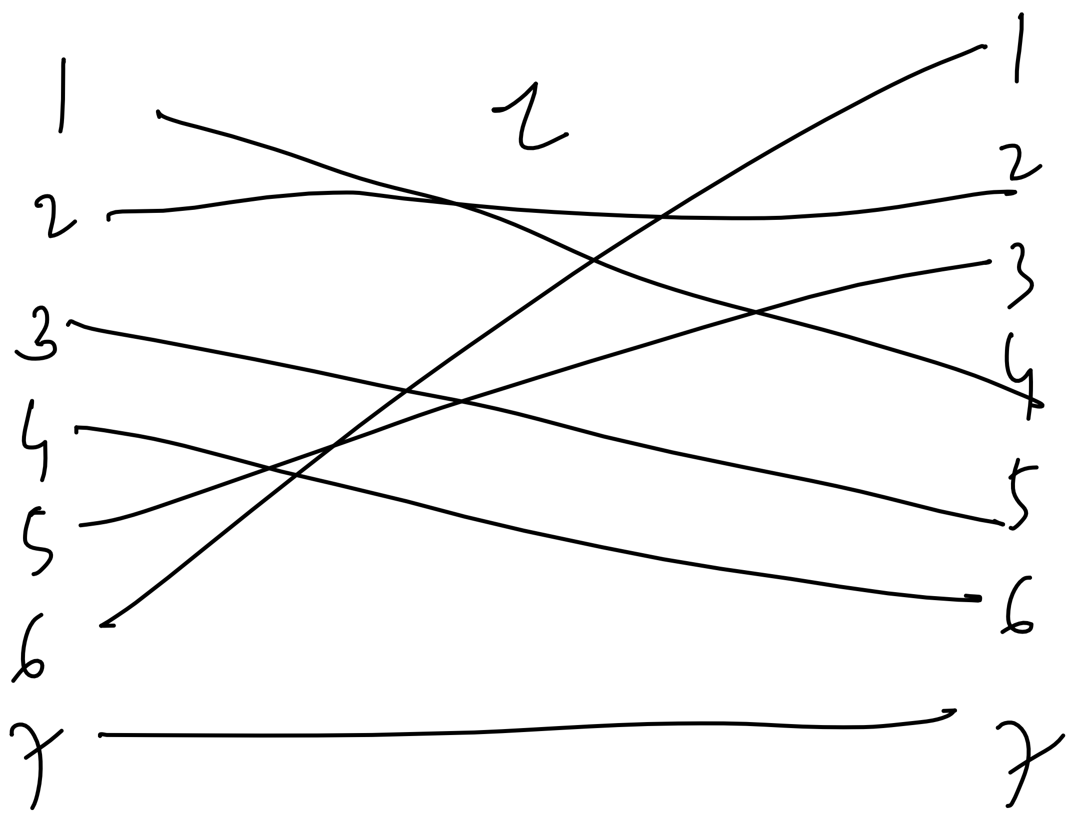
$\tau_1 = \tau_2$

S_4 = Set of all permutations on $\{1, 2, 3, 4\}$



$(132)(4)$

(132)



$(1\ 4\ 6)(2)(3\ 5)(7)$

* Every permutation is a product of disjoint cycles.

$\sigma = (1\ i\ j\ k\ l\ m)(\quad)$

$(1\ 2\ 3)$

Transposition : A cycle of order 2 is called a transposition

$(1\ 2)\quad(3\ 5)\quad(3\ 9)$

Every cycle is a product of transpositions.

$(a_1 a_2 \cdots a_k)$

$(1\ 2\ 3) = (1\ 3)(1\ 2)$

$(1\ 3)(1\ 2)$

$$\begin{aligned}(1) &= 2\\ (2) &= 3\\ (3) &= 1\end{aligned}$$

$(1\ 2\ 3)$

$$\begin{aligned}(1) &\to 2\\ (2) &\to 3\\ (3) &\to 1\end{aligned}$$

$(a_1 a_2 \cdots a_n) = (a_1 a_n)(a_1 a_{n-1}) \cdots (a_1 a_2)$

$(1\ 2\ 3) = (1\ 2)(2\ 3)$

$f \circ g\,(a) = f\,(g(a))$

* Every permutation is a product of transpositions

$\sigma = (\quad\quad)(\quad\quad) - -$

cycles cycles

$(a_1, a_2 \cdots a_n) = (a_1\, a_n)(a_1\, a_{n-1}) - - - (a_1\, a_2)$

$= (a_1 a_2)(a_2 a_3) - - - (a_{n-1}\, a_n)$

$(1\,2\,3) = (1\,2)(2\,3)$ ✓

$= (1\,3)(1\,2)$ ✓

$(1\,3) = (1\,2)(2\,3)(1\,2)$

$1 \to 3$

$2 \to 2$

$3 \to 1$

$id = (1\,2)(1\,2)$

$= (1\,2)(1\,2)(3\,4)(3\,4)$ $1 \to 1$, $2 \to 2$

Even Permutation : A permutation σ is said to be even if the no of transpositions required to express σ is even

odd Permutation : ✓

$f \circ g \neq g \circ f$?

id permutation is even

$id = t_1 t_2 - - t_{m-1} t_m$

Claim m is even

$t_{m-1} t_m = (a\,b)(c\,d)$

$= (a\,b)(a\,b)$

$= (a\,c)(c\,b)$

$= (a\,b)(b\,c)$

$(a\,b) \circ (c\,d) = (c\,d) \circ (a\,b)$