Mathematical Modelling of Ultrasonic Machining

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25th March, 2012

Abstract

A model is proposed for the mechanism of material removal in ultrasonic machining. An analysis of material removal in ultrasonic machining due to (a)direct impact of tool on the abrasive grains in contact with the workpiece and tool, and (b)impact of grains accelerated by the vibrating tool is presented. From this model and fundamental physical principles, material removal rate is derived.

Keywords: Ultrasonic machining,direct impact,abrasive grains,material removal rate.

1 Introduction

Ultrasonic machining is used for machining hard materials and brittle materials.Ultrasonic machining (USM) is the removal of material by the abrading action of grit-loaded liquid slurry circulating between the workpiece and a tool vibrating perpendicular to the workface at a frequency above the audible range.The workpiece is placed under the face of the tool which is subjected to high frequency vibration perpendicular to the surface being machined. Abrasive slurry is conveyed to the working zone between the face of the tool and the surface being machined. The tool moves towards the workpiece and is subjected to a static driving force. Repetitive impact of the tool on the grains of the abrasive material lead to the fracture of the workpiece material and to the creation of a cavity with the shape mirror formed of the tool. The abrasive particles are propelled or hammered against the workpiece by the transmitted vibrations of the tool. The particles then microscopically

erode the workpiece.
Miller assumed that the material removal rate depends upon work-hardening and plastic deformation of the workpiece. His analysis is mostly applicable to plastic materials, however, most of the materials machined by ultrasonic machining are brittle. He also assumed that all the grains are cubes of the same size and that all the grains take part in cutting.

2 Working Principle

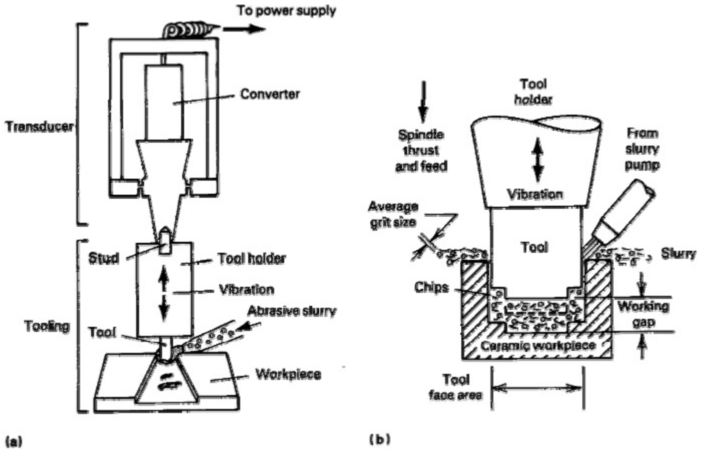

Fig 1. Schematic showing key component of a typical USM installation (a)Transducer assembly coupled to tooling assembly of unit (b)Close-up view of tooling assembly being used to machine a ceramic.

In ultrasonic machining, a tool of desired shape vibrates at an ultrasonic frequency (19 - 25 kHz) with an amplitude of around 15 - 50 μm over the workpiece. Generally the tool is pressed downward with a feed force. Between the tool and workpiece, the machining zone is flooded with hard abrasive particles generally in the form of a water based slurry. As the tool vibrates over the workpiece,these vibrations are transmitted to the abrasive particles in the slurry via an energy focusing device or horn/tool assembly. The abrasive particles act as the indenters and indent both the work material and the tool. The abrasive particles, as they indent, the work material, would remove the same, particularly if the work material is brittle, due to crack initiation, propagation and brittle fracture of the material. A constant stream of abrasive slurry passes between the tool and the work to provide abrasives and carry away fractured particles / chips. Hence, USM is mainly used for machining brittle materials.

3 Analysis of Material Removal Rate

Material removal during USM due to cavitation under the tool and chemical corrosion due to slurry media are considered insignificant. Hence, material removal due to these two factors has been ignored. Contributions to the material removal by abrasive particles due to 'throwing' and 'hammering' actions have been analysed.

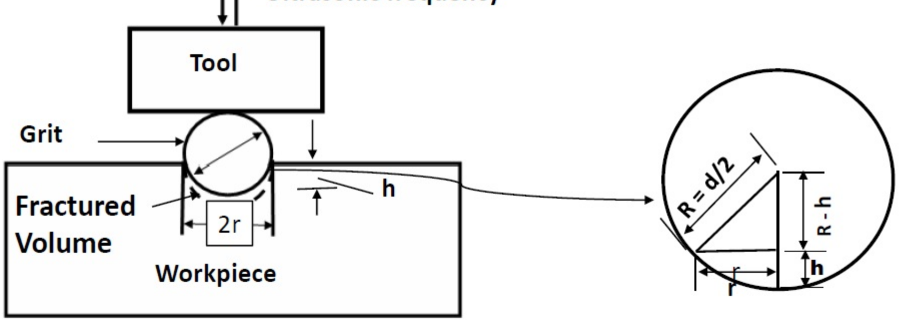

Fig 2. Development of fracture in the workpiece due to hitting by an abrasive by hammering.

Abrasive particles are assumed to be spherical in shape having diameter as 'd' units. There are two possibilities when the tool hits an abrasive particle[M.C.Shaw(1956)]. If the size of the particle is small and the gap between the bottom of the tool and work surface is large enough, then the particle will be thrown by the tool, to hit the work surface(throwing model). Under the reverse conditions, the particle will be hammered over the work surface. In the both cases, a particle after hitting the work surface generates a crater of depth 'h' and radius 'r'. It is also assumed that the volume of the particle removed is approximately proportional to the diameter of indentation (2r). The volume of material (Vg) removed (shown by dashed lines in Fig 2, assuming hemi-spherical crater) due to fracture cycle is given by

$$V_g = (1/2)((4/3)\pi r^3) \tag{1}$$

From Fig.2, it can be shown that

$$r^2 = (d/2)^2 - ((d/2) - h)^2$$

neglecting h^2 terms as h≪d

$$r^2 \approx dh \tag{2}$$

From Eqs. (1) and (2), we can write

$$V_g = K_1(hd)^{3/2} \tag{3}$$

where, K_1 is a constant.

Number of impacts (N) on the workpiece by the grits in each cycle will depend upon the number of grits beneath the tool at any time. This is inversely proportional to the diameter of the grit (assumed spherical) as given below.

$$N = K_2(1/d^2) \tag{4}$$

where, K_2 is a constant of proportionality.

All abrasive particles under the tool need not be necessarily effective. Let K_3 be the probability of an abrasive particle under the tool being effective. Then volume (V) of material removed per second will be equal to frequency (f) times the amount of material removed per cycle (V_c).

$$V = V_c \times f = K_1 K_2 K_3 \sqrt{h^3/d)} \times f \tag{5}$$

To evaluate the depth of penetration 'h' of an Abrasive particle, Shaw [1956] proposed two models. Model 1 considers that when a particle is hit by the tool it is thrown (Fig.3) on the workpiece surface. Model 2 assumes that a particle is hammered (Fig.2) by the tool into the workpiece.

3.1 Model 1 (Grain Throwing Model)

It is assumed that a particle is hit and thrown by tool onto the workpiece surface. Assuming sinusoidal vibration, the displacement (Y) of the tool is given by Eq.(6) in which 't' is time period and a/2 is amplitude of oscillation.

$$Y = (a/2)\sin(2\pi f t) \tag{6}$$

From Eq.(6), velocity of the tool is evaluated as follows

$$\dot{Y} = \pi a f \cos(2\pi f t) \tag{7}$$

The maximum velocity of tool $\dot{Y}_{max}$ is derived as follows (for $\cos(2\pi f t)$=1):

$$\dot{Y}_{max} = \pi a f \tag{8}$$

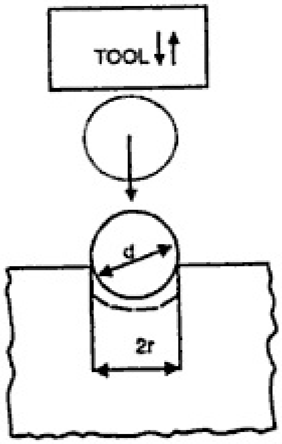

Fig 3. Throwing.

let us assume that the grits also leave the tool with the same maximum velocity,i.e.$\dot{Y}_{max}$. Then (KE) of a grit is given by

$$KE = (1/2)m\pi^2a^2f^2$$

$$KE = (1/2)((\pi/6)d^3\rho_a)\pi^2a^2f^2 \tag{9}$$

A grit penetrates to the depth equal to 'h' into the workpiece. It is assumed that full KE of the grit is absorbed by the workpiece before it comes to rest. Then the work done (W_g) by a grit (assuming triangular variation of force (F) with the depth of penetration) is given by (From Fig.4)

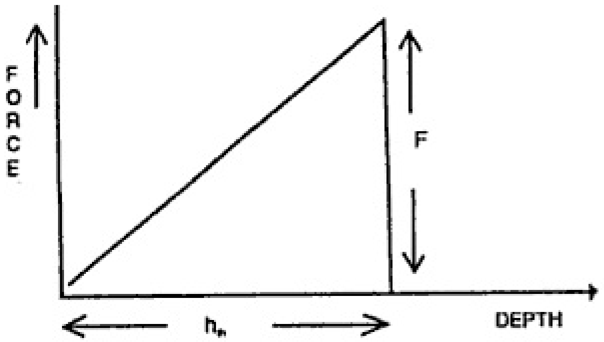

Fig 4. Variation of force (F) with a change in depth of penetration.

$$W_g = (1/2)Fh_{th} \tag{10}$$

Work done by the grit (W_g) should be equal to the KE of the particle.

$$(1/2)Fh_{th} = (1/2)((\pi/6)d^3\rho_a)\pi^2a^2f^2$$

$$h_{th} = (\pi^3a^2f^2d^3\rho_a)/6F \tag{11}$$

'F' can be written in terms of workpiece property that can be known beforehand. Mean stress acting on the workpiece surface (σ_w) is given by(Using Eq.(2), and taking h=h_{th})

$$\sigma_w = F/A = F/(\pi h_{th}d)$$

$$F = \pi\sigma_w h_{th}d \tag{12}$$

From Eqs.(11) and (12),

$$h_{th} = (\pi^3a^2f^2d^3\rho_a)/(6\pi\sigma_w h_{th}d)$$

$$h_{th}^2 = (\pi^2a^2f^2d^2\rho_a)/6\sigma_w$$

$$h_{th} = \pi afd\sqrt{\rho_a/6\sigma_w} \tag{13}$$

Volumetric material removal rate due to throwing mechanism (V_{th}) can be obtained using Eqs.(5) and (13).

$$V_{th} = K_1K_2K_3(\pi^2a^2\rho_a/6\sigma_w)^{3/4}df^{5/2} \tag{14}$$

3.2 Model 2 (Grain Hammering Model)

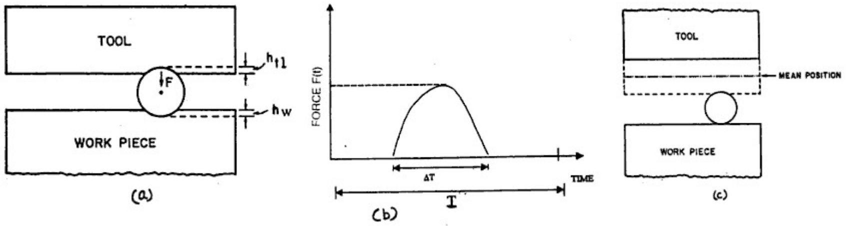

Fig.5. (a)Partial penetration of a grit in the tool and workpiece, (b)Variation of force(F) with time(T), (c)Schematic diagram if a grain hammering model.

When the gap between the tool and the workpiece is smaller than the diameter of the grit it will result into partial penetration in tool (h_{tl}) as well as in the workpiece (h_w)(Fig.5a). The values of h_w and h_{tl} will depend on the hardness of the tool and workpiece material, respectively. Force 'F' acts on the abrasive particle only for a short time(ΔT) during the cycle time ’T’(Fig.5b). During this time period, the abrasive particle is in contact with the tool and the workpiece both(Fig.5c). The mean force(F_{avg}) on the grit can be expressed by Eq.(15)

$$F_{avg} = (1/T) \int_0^T F(t)dt \tag{15}$$

Here, F(t) is the force at any instant of time ’t’. Force on the grit by the tool starts increasing as soon as the grit gets in contact with both the tool and the workpiece at the same time. It attains maximum value and then starts decreasing until attains zero value. Hence, the momentum equation can be written as

$$\int_0^T F(t)dt \approx (F/2)\Delta T \tag{16}$$

Total penetration due to hammering (h_h) (Fig.5(a)) is given as

$$h_h = h_w + h_{tl} \tag{17}$$

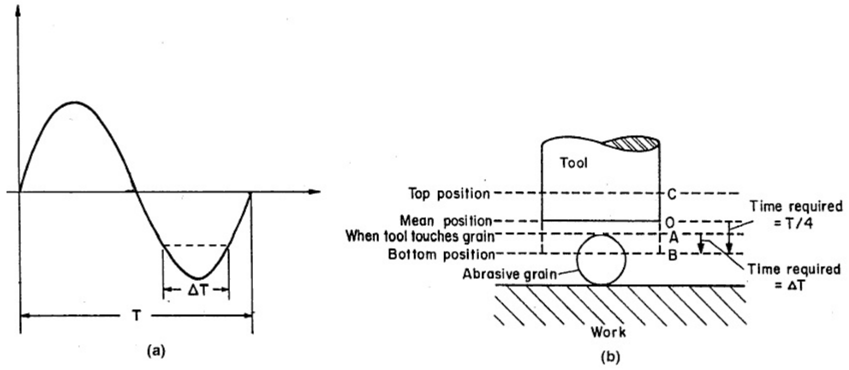

Fig.6 (a) Assumed mode of tool vibration. (b) Various positions of the tool while hitting workpiece via a grit.

a/2 is amplitude of oscillation of the tool. The mean velocity of the tool during the quarter cycle(from O to B in Fig.6) is given by (a/2)/(T/4).Therefore, time(ΔT) required to travel from A to B is given by the following equation:

$$\Delta T = (h_h/(a/2)) \cdot (T/4)$$

$$\Delta T = (h_h/a)(T/2) \tag{18}$$

From Eqs. (15),(16) and (18)

$$F = F_{avg}(4a/h_h) \tag{19}$$

Let 'N' be the number of grains under the tool. Stress acting on the tool (σ_{tl}) and the workpiece (σ_w) can be found as follows:

$$\sigma_w = F/(N(\pi h_w d)) \tag{20}$$

$$\sigma_{tl} = F/(N(\pi h_{tl} d))$$

From Eq.(20)

$$\sigma_{tl} = \sigma_w(h_w/h_{tl}) \tag{21}$$

From Eqs.(4),(19) and (20)

$$\sigma_w = F_{avg}(4ad^2/(h_h K_2(\pi h_w d)))$$

$$\sigma_w = 4F_{avg}ad/(\pi K_2 h_w(h_w + h_{tl}))$$

$$\sigma_w = 4F_{avg}ad/(\pi K_2 h_w^2((h_{tl}/h_w) + 1)) \tag{22}$$

From Eq.(21)

$$h_{tl}/h_w = \sigma_w/\sigma_{tl} = j \tag{23}$$

j can be taken as the ratio of hardness of workpiece material to the hardness of tool material. From Eqs.(22) and (23),

$$h_w = \sqrt{4F_{avg}ad/(\sigma_w \pi K_2(j+1))} \tag{24}$$

Volume material removal rate from the workpiece due to hammering mechanism (V_h), can be evaluated using Eqs.(5) and (24) as follows:

$$V_h = K_1 K_2 K_3[4aF_{avg}/(\sigma_w \pi K_2(j+1))]^{3/4} df \tag{25}$$

From computational results obtained using Eqs.(14) and (25), it is observed that $V_h \gg V_{th}$

4 Result and Discussion

Fig.7 shows the effect of amplitude of vibration on MRR. Increase in the amplitude of vibration increases MRR. This is the most significant process parameter that affects MRR. Under certain circumstances, this also limits the maximum size of the abrasive to be used.

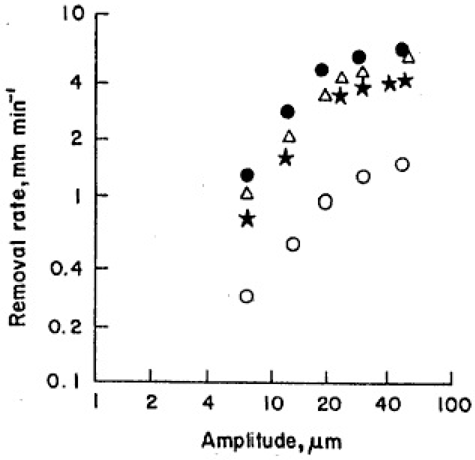

Fig. 7 Effects of amplitude of vibration on material removal rate during USM. Workpiece: glass; tool: steel; abrasive: B_4 C (120 mesh size); pressure: • 0.20 MPa; Δ 0.16 MPa; * 0.10 MPa; ○ 0.04 MPa. [*Kremer et al.*, 1981].

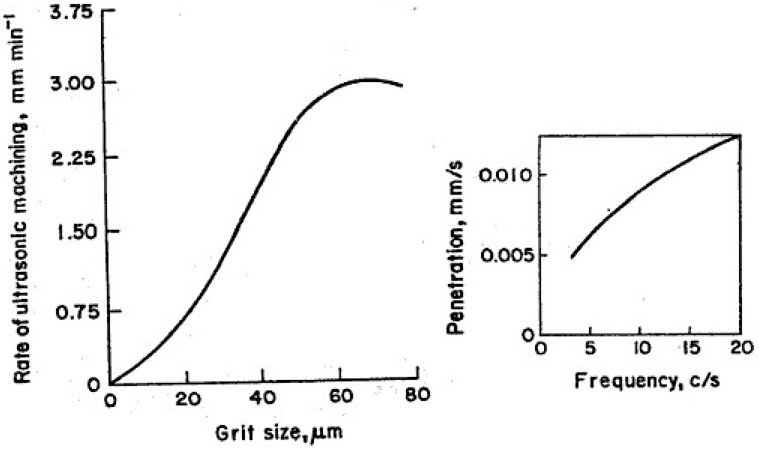

Fig.8 Relationship between (a) penetration rate and abrasive grain size [Kazantsev, 1956]. (b) penetration rate and frequency of vibration of tool [Neppiras and Foskett,1956].

The effect of abrasive grain size is illustrated in Fig.8(a). An increase in abrasive grain size results in higher MRR but poorer surface finish. Surface finish is also influenced by the parameter like amplitude of vibration, properties of the workpiece material, finish of the tool surface and viscosity of the liquid carrier for the abrasives. Maximum MRR is achieved when abrasive grain size is comparable with the amplitude of vibration of the tool. Hardness of the abrasives and method of introducing slurry in the machining zone also affect the machining rates. Frequency of vibration also has a significant effect on MRR(Fig.8(b)). Efficient cutting is obtained at resonance frequency will yield higher MRR provided machining is done at the resonance frequency. MRR goes down as depth of hole increases. It happens so because of inefficient flow of slurry through the cutting zone at high depth.

5 Conclusion

It is clear from the derived Eqs. that the material Removal rate(MRR) depends on the amplitude and frequency of vibration, static force, size of abrasive particle and property of tool and workpiece. MRR in the hammering model is more than the throwing model. It is clear from the graph that MRR increases as the amplitude of vibration increases upto an optimum value,beyond this it starts decreasing. MRR also increases as the frequency of vibration and pressure increases. As the grit size increases MRR also increase upto an optimum value after that it stars deceasing.

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