Lab Assignment II

1. We examine the behaviour of error as the degree of interpolating polynomial increases for equispaced interpolation points. We construct a polynomial of degree $\leq n$ on $[-1,1]$ in *Lagrange's form* using $n+1$ equispaced points for the Runge function $f(x) = 1/(1+25x^2)$. The interpolating data are (x_i, y_i) where $x_i = 2i/n - 1$ and $y_i = f(x_i)$ $(i = 0, 1, \cdots, n)$.

   Next calculate the error $||f - p_n||_\infty = \max_{x\in[-1,1]} |f(x) - p_n(x)|$. To implement this, we take large number of points in $[-1,1]$. If we take $m = 1001$ points, then $z_i = 2i/m - 1$ and we approximate the above norm by $||f - p_n||_\infty = \max_{0\leq i\leq m} |f(z_i) - p_n(z_i)|$. Note that in case z_i concides with any x_j, then $|f(z_i) - p_n(z_i)| = 0$. We also find z_i at which this maximum occurs. Due to symmetry, it occurs at left as well as right ends.

   Finally we output the data corresponding to z_i $(i = 0, 1, \cdots, m)$ in a file (say prunge.dat). This data file has three columns with first, second and third columns containing z_i, $f(x_i)$ and $p_n(z_i)$. Using gnuplot, you can plot them using the following command

```
plot 'prunge.dat' u 1:2 w l ls 2 title "f(x)",'prunge.dat' u 1:3 w l ls 3 title "p_{10}(x)"
```

   for $n = 10$. (Here 'u' stands for 'using', '1:2' stands for 'columns 1 and 2', 'l' for 'lines' and 'ls' for 'linestyle'. Syntax for your desktop may be different.)

   Your program should handle upto interpolating polynomial of degree 50. Here is a typical input/output

```
Enter degree n of interpolating polynomial: 9
||f-P_9|| = 0.3003 occurs at x=0.9280
```

   Also, a screenshot of the gnuplot command is the following: