Lab Assignment III

1. We examine the behaviour of error with the distribution of interpolation points. We construct a polynomial of degree $\leq n$ on $[a, b]$ in *Newtons's form* using $n+1$ points. The interpolating data are (x_i, y_i), where $x_i = g(i, a, b, n)$ and $y_i = f(x_i)$ $(i = 0, 1, \cdots, n)$. If we take $g(i, a, b, n) = a + (b - a)i/n$, then we get equispaced points. On the other hand,

$$g(i, a, b, n) = \frac{1}{2}\left(a + b + (a - b)\cos\left(\frac{2i+1}{2n+2}\pi\right)\right)$$

give Chebyshev points. Note that Chebyshev points are generated in increasing order.

For f, we take the Runge function $f(x) = 1/(1 + 25x^2)$.

The divided differences used for the polynomial need to be stored in the array for y which initially contain the function values at the interpolation points.

Next calculate the error $||f - p_n||_\infty = \max_{x \in [a,b]} |f(x) - p_n(x)|$. To implement this, we take large number of points in $[a, b]$. If we take $m + 1$ points with $m = 1000$, then $z_i = a + (b - a)i/m$ $(i = 0, 1, \cdots, m)$ and we approximate the above norm by $||f - p_n||_\infty = \max_{0 \leq i \leq m} |f(z_i) - p_n(z_i)|$. We also find z_i at which this maximum occurs. Due to symmetry, it occurs at left as well as right ends in case of $a + b = 0$.

The evaluation of the interpolation polynomial at each z_i must be done using nested multiplication technique.

Finally we output the data corresponding to z_i $(i = 0, 1, \cdots, m)$ in a file (say eqp.dat/chp.dat for equispaced/Chebyshev). These data file have three columns with first, second and third columns containing z_i, $f(x_i)$ and $p_n(z_i)$. Using gnuplot, you can plot them using the following command

```
plot 'eqp.dat' u 1:2, 'eqp.dat' u 1:3,'chp.dat' u 1:3
```

Your program should handle upto interpolating polynomial of degree 50. Here are typical input/output:

```
Enter a and b: -1 1
Enter degree n of interpolating polynomial: 10
Enter 0/1 for equispaced/Chebyshev points: 0
||f-P_10|| = 1.9156 occurs at x=0.9400


Enter a and b: -1 1
Enter degree n of interpolating polynomial: 10
Enter 0/1 for equispaced/Chebyshev points: 1
||f-P_10|| = 0.1091 occurs at x=-0.1560
```

Also, a screenshot of the gnuplot command is the following:

Legend:
- 'eqout.dat' u 1:2
- 'eqout.dat' u 1:3
- 'chout.dat' u 1:3