Lab Assignment V

Given the interpolation points $(x_i, f_i), i = 0, 1, 2, \cdots, n$ $(a = x_0 < x_1 < x_2 < \cdots < x_n = b)$, a cubic spline is given by

$$S(x) = \Big\{S_i(x), x \in [x_i, x_{i+1}]\Big\}, i = 0, 1, 2, \cdots, n-1,$$

where S_i is a polynomial of degree less equal to 3. Each S_i is given by

$$S_i(x) = A_i(x_{i+1} - x)^3 + B_i(x - x_i)^3 + C_i(x - x_i) + D_i(x_{i+1} - x), \qquad i = 0, 1, \cdots, n-1,$$

where

$$A_i = \frac{z_i}{6h_i}, \quad B_i = \frac{z_{i+1}}{6h_i}, \quad C_i = \left(\frac{f_{i+1}}{h_i} - \frac{z_{i+1}h_i}{6}\right), \quad D_i = \left(\frac{f_i}{h_i} - \frac{z_i h_i}{6}\right),$$

where $h_i = x_{i+1} - x_i, i = 0, 1, 2, \cdots, n-1$. For natural cubic spline $z_0 = z_n = 0$ and $z_i, i = 1, 2, \cdots, n-1$ are the solution of tridiagonal system

$$\begin{bmatrix} d_1 & h_1 & & & & \\ h_1 & d_2 & h_2 & & & \\ & h_2 & d_3 & h_3 & & \\ & & \ddots & \ddots & \ddots & \\ & & & h_{n-3} & d_{n-2} & h_{n-2} \\ & & & & h_{n-2} & d_{n-1} \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ \vdots \\ z_{n-2} \\ z_{n-1} \end{bmatrix} = \begin{bmatrix} r_1 \\ r_2 \\ r_3 \\ \vdots \\ r_{n-2} \\ r_{n-1} \end{bmatrix},$$

where $d_i = 2(h_i + h_{i-1})$, $r_i = 6(b_i - b_{i-1})$ and $b_i = (f_{i+1} - f_i)/h_i$. This can be solved by LU decomposition.

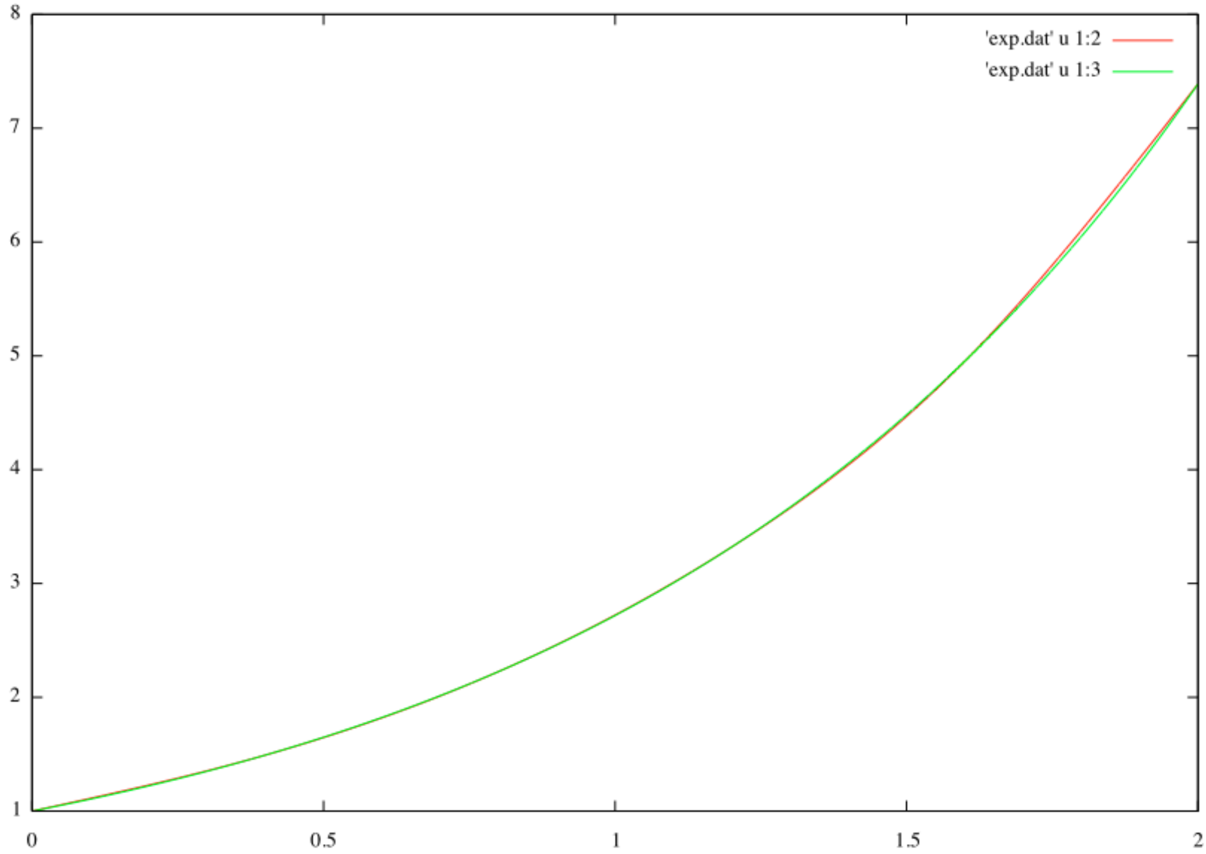
Your program should read n, a, b from the keyboard. Then $n + 1$ interpolation data (x_i, f_i) for $i = 0, 1, 2, \cdots, n$ are generated from $x_i = a + (b - a)i/n$ and $f_i = f(x_i)$. The following data are for $a = 0, b = 2, n = 5$ and $f(x) = e^x$.

Below is a typical input/output:

```
Enter n:5
Enter x_0 and x_n: 0 2
z[0]  0.000000
z[1]  1.701545
z[2]  2.264751
z[3]  2.771691
z[4]  6.836216
z[5]  0.000000
```

To find the interpolation value corresponding to any $x \in [a, b]$, find i such that $x \in [x_i, x_{i+1}]$. Then we find the value from the corresponding $S_i(x)$. Now we take $m + 1$ points with $m = 1000$, then $t_i = a + (b - a)i/m$ $(i = 0, 1, \cdots, m)$. We print the data corresponding to t_i $(i = 0, 1, \cdots, m)$ in a file (say exp.dat) which have three columns with first, second and third columns containing t_i, $S(t_i)$ and $f(t_i)$. Using gnuplot, you can plot them using the following command (the command may be different for gnuplot installed in your desktop.) A screenshot is attached.

```
plot 'exp.dat' u 1:2 w l ls 1,'exp.dat' u 1:3 w l ls 2
```


'exp.dat' u 1:2
'exp.dat' u 1:3
8
7
6
5
4
3
2
1
0
0.5
1
1.5
2