Lab Assignment VI

Consider the boundary value problem (BVP)

$$\frac{d^2u}{dx^2} = f(x), \qquad u(0) = A,\ u(1) = B$$

Let x_i, $i = 0, 1, \cdots, n+1$ be $n+2$ equispaced grid points with $x_i = ih$, where $h = 1/(n+1)$. Denoting $u_i \approx u(x_i)$, the above system (upon discretization) can be reduced to

$$\begin{bmatrix} -2 & 1 & & & & \\ 1 & -2 & 1 & & & \\ & 1 & -2 & 1 & & \\ \vdots & \vdots & \vdots & \vdots & \vdots & \\ & & & & -2 & 1 \\ & & & & 1 & -2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ \vdots \\ u_{n-1} \\ u_n \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ \vdots \\ d_{n-1} \\ d_n \end{bmatrix},$$

where $d_1 = h^2 f(x_1) - u(0)$, $d_i = h^2 f(x_i)$ for $i = 2, \cdots, n-1$ and $d_n = h^2 f(x_n) - u(1)$.

Solve the above linear system using Gauss-Seidel iteration for $f(x) = 6x, A = 0, B = 2$ for which the exact solution is $u(x) = x + x^3$. Instead of using a full matrix, you may use three arrays to store the nonzero elements of the matrix and one array to store the right hand side. Iteration is stopped when $||u^{(k)} - u^{(k-1)}||_\infty < \epsilon$ where choose $\epsilon = 10^{-4}$. Initial components of $u^{(0)}$ are zero. You need two arrays in the algorithm to store the previous iteration values for the norm calculation. Your input/output should be in the following format where $---$ contains the output from your code.

```
Here n+2 is the total number of grid points
Enter n:9
No. of iterations : --
u[1]   ---
u[2]   ---
u[3]   ---
u[4]   ---
u[5]   ---
u[6]   ---
u[7]   ---
u[8]   ---
u[9]   ---
```