Lab Assignment VI

Consider the boundary value problem (BVP)

$$\frac{d^2u}{dx^2} = f(x), \qquad u(0) = A,\ u(1) = B$$

Let x_i, $i = 0, 1, \cdots, n+1$ be $n+2$ equispaced grid points with $x_i = ih$, where $h = 1/(n+1)$. Denoting $u_i \approx u(x_i)$, the above system (upon discretization) can be reduced to

$$
\begin{bmatrix}
-2 & 1 & & & & & \\
1 & -2 & 1 & & & & \\
& 1 & -2 & 1 & & & \\
\vdots & \vdots & \vdots & \vdots & \vdots & & \\
& & & & -2 & 1 \\
& & & & 1 & -2
\end{bmatrix}
\begin{bmatrix}
u_1 \\ u_2 \\ u_3 \\ \vdots \\ u_{n-1} \\ u_n
\end{bmatrix}
=
\begin{bmatrix}
d_1 \\ d_2 \\ d_3 \\ \vdots \\ d_{n-1} \\ d_n
\end{bmatrix},
$$

where $d_1 = h^2 f(x_1) - u(0)$, $d_i = h^2 f(x_i)$ for $i = 2, \cdots, n-1$ and $d_n = h^2 f(x_n) - u(1)$.

Solve the above linear system using SOR method for $f(x) = 6x, A = 0, B = 2$ for which the exact solution is $u(x) = x + x^3$. Instead of using a full matrix, you may use three arrays to store the nonzero elements of the matrix and one array to store the right hand side. Iteration is stopped when $||u^{(k)} - u^{(k-1)}||_\infty < \epsilon$ where choose $\epsilon = 10^{-4}$. Initial components of $u^{(0)}$ are zero. You need two arrays in the algorithm to store the previous iteration values for the norm calculation. Your input/output should be in the following format where $---$ contains the output from your code.

```
Here n+2 is the total number of grid points
Enter n:9
Enter relaxation parameter w :1.5
No. of iterations : --
u[1]    ---
u[2]    ---
u[3]    ---
u[4]    ---
u[5]    ---
u[6]    ---
u[7]    ---
u[8]    ---
u[9]    ---
```