

## Assignment-III

1. Find the solution of the modified single dose model:

$$\begin{aligned}\frac{dx}{dt} &= -k_1x, & x(0) &= x_0, \\ \frac{dy}{dt} &= k_1x - k_2(x - y), & y(0) &= 0.\end{aligned}$$

Also, discuss the long-term behaviour of the solution.

2. In fluid mechanics, the Reynolds number is a dimensionless number involving the fluid speed  $U$ , density  $\rho$ , dynamic viscosity  $\mu$  and a characteristic length  $L$ . Find the Reynolds number.

3. Determine whether the equation

$$\frac{dE}{dt} = \left[ mr^2 \left( \frac{d^2\theta}{dt^2} \right) + mgr \sin \theta \right] \frac{d\theta}{dt}$$

is compatible, where  $E$  is the energy.

4. Find how the centrifugal force  $F$  of a particle depends on its mass  $m$ ; velocity  $v$  and the radius  $r$  of the curvature of its path.
5. The speed of sound  $v$  in a gas depends on the pressure  $p$  and the density  $\rho$ . Find how the speed  $v$  depends on  $p$  and  $\rho$ .
6. Let  $P$  be the pressure drop per unit length in pipe flow. If  $P$  depends on the diameter  $D$  of the pipe, mean velocity  $v$  of the flow, the density  $\rho$  and dynamic viscosity  $\mu$  of the fluid, then express  $P$  as function of the rest of the variable.
7. Find a model that determines the terminal velocity of a raindrop falling from a motionless cloud. You may assume that the variables that affect the terminal velocity  $v$  are the size of the raindrop  $r$  (assuming a raindrop to be a sphere of radius  $r$ ), the density  $\rho$  of the raindrop, and the air viscosity  $\mu$ . (Ideally, it depends on the density difference of the falling body and the surrounding medium. Since air density is negligible compared to that of the raindrop, the density difference is approximately the density of the raindrop.)
8. Assume that the force  $F$  opposing the fall of a raindrop through air depends on the dynamic viscosity  $\mu$ , velocity  $v$  and the diameter  $r$  of the raindrop. Assuming that the effect of density can be ignored, find the relation of  $F$  with the rest of the governing variables.
9. Consider an experiment in which a pendulum bob, of mass  $m$  and length  $l$ , is released from rest from an angle  $\theta_0$ . Let  $\tau$  be the period of the oscillation.

- a. If the drag force is  $R$ , then show that

$$\tau = \sqrt{l/g} h \left( \theta_0, \frac{R}{mg} \right).$$

- b. Suppose the drag force  $R$  is proportional to  $v^2$ , where  $v$  is the velocity. If  $R = -kv^2 \text{sgn}(v)$ , then show that

$$\tau = \sqrt{l/g} h \left( \theta_0, \frac{kl}{m} \right).$$

10. With the help of Pi theorem, find the solution of the diffusion equation

$$\frac{\partial u}{\partial t} = D \frac{\partial^2 u}{\partial x^2}, \quad 0 < x < \infty, t > 0$$

subject to the boundary conditions

$$u(0, t) = u_0, \quad u(\infty, 0) = 0, \quad t > 0,$$

and initial conditions

$$u(x, 0) = 0, \quad 0 < x < \infty.$$

(Imagine a long pipe occupies  $0 \leq x < \infty$  and it is filled with clean water at  $t = 0$ . A source of constant concentration  $u_0$  is introduced at  $x = 0$ . The governing equation models the concentration  $u(x, t)$  at position  $x$  and time  $t > 0$ .)

11. Fit the model  $\hat{y} = cx^2$  to the data  $(1, 10), (3, 90), (5, 250), (7, 495)$  using Chebyshev's criterion to minimize the largest deviation.