

Multivariate Analysis with R

Estimation of Parameters

:::

Maximum Likelihood Estimation

Shalabh

Department of Mathematics and Statistics

Indian Institute of Technology Kanpur

Maximum Likelihood Estimation: Multivariate Normal Distribution

μ is unknown and Σ is unknown:

$$\hat{\underline{\mu}}_{MLE} = \underline{\bar{x}} = \begin{pmatrix} \bar{x}_1 \\ \bar{x}_2 \\ \vdots \\ \bar{x}_p \end{pmatrix}$$

R command: `colMeans (x)`

$$\hat{\Sigma}_{MLE} = \frac{1}{n} \sum_{\alpha=1}^n (\underline{x}_{\alpha} - \underline{\bar{x}})' (\underline{x}_{\alpha} - \underline{\bar{x}})$$

R command: `var (x)` computes $\frac{1}{n-1} \sum_{\alpha=1}^n (\underline{x}_{\alpha} - \underline{\bar{x}})' (\underline{x}_{\alpha} - \underline{\bar{x}})$

$\frac{1}{n} \sum_{\alpha=1}^n (\underline{x}_{\alpha} - \underline{\bar{x}})' (\underline{x}_{\alpha} - \underline{\bar{x}})$ is computed by `(n-1) * var (x) / n`

Maximum Likelihood Estimation: Random Vectors and Sample Mean in Multivariate Normal Distribution with R

```
library(mvtnorm)
```

```
x = rmvnorm(n=5,mean=c(10,20,30),  
sigma=diag(c(2,3,4)))
```

```
> x
```

	[,1]	[,2]	[,3]
[1,]	10.205021	22.21307	30.14057
[2,]	7.732032	16.75653	28.33492
[3,]	8.538400	15.54037	30.62831
[4,]	10.569876	20.13748	33.38929
[5,]	12.036105	22.94432	30.54506

```
> colMeans(x)
```

```
[1] 9.816287 19.518355 30.607630
```

Maximum Likelihood Estimation: Covariance Matrix of Multivariate Normal Distribution in R

```
> var(x)
```

```
      [,1]      [,2]      [,3]  
[1,] 2.905930  4.989717  1.621563  
[2,] 4.989717 10.708507  1.610952  
[3,] 1.621563  1.610952  3.281331
```

Maximum Likelihood Estimation: Covariance Matrix of Multivariate Normal Distribution in R

```
> n=ncol(x)
```

```
> n
```

```
[1] 3
```

```
> (n-1)*var(x)/n
```

	[,1]	[,2]	[,3]
[1,]	1.937286	3.326478	1.081042
[2,]	3.326478	7.139005	1.073968
[3,]	1.081042	1.073968	2.187554

Maximum Likelihood Estimation: Correlation Matrix of Multivariate Normal Distribution in R

```
> cor(x)
```

	[,1]	[,2]	[,3]
[1,]	1.0000000	-0.2303695	0.3300610
[2,]	-0.2303695	1.0000000	-0.1484404
[3,]	0.3300610	-0.1484404	1.0000000

Maximum Likelihood Estimation: Multivariate Normal Distribution

```
R Console
> library(mvtnorm)
> x = rmvnorm(n=5, mean=c(10,20,30), sigma=diag(c(2,3,4)))
> x
      [,1]      [,2]      [,3]
[1,] 10.205021 22.21307 30.14057
[2,]  7.732032 16.75653 28.33492
[3,]  8.538400 15.54037 30.62831
[4,] 10.569876 20.13748 33.38929
[5,] 12.036105 22.94432 30.54506
> colMeans(x)
[1]  9.816287 19.518355 30.607630
> var(x)
      [,1]      [,2]      [,3]
[1,] 2.905930  4.989717  1.621563
[2,] 4.989717 10.708507  1.610952
[3,] 1.621563  1.610952  3.281331
> n=ncol(x)
> n
[1] 3
> (n-1)*var(x)/n
      [,1]      [,2]      [,3]
[1,] 1.937286  3.326478  1.081042
[2,] 3.326478  7.139005  1.073968
[3,] 1.081042  1.073968  2.187554
> cor(x)
      [,1]      [,2]      [,3]
[1,] 1.0000000 0.8944764 0.5251293
[2,] 0.8944764 1.0000000 0.2717648
[3,] 0.5251293 0.2717648 1.0000000
> |
```