ESO 209: PROBABILITY & STATISTICS
Session: 2009-10 Semester: II
(Instructor: S. Mitra)

Problem Set 2

[1] For events A, B and C such that $P(C) > 0$, prove that

(a) $P(A \cup B \mid C) = P(A \mid C) + P(B \mid C) - P(AB \mid C)$

(b) $P(A^C \mid C) = 1 - P(A \mid C)$.

[2] Let A and B be two events such that $0 < P(A) < 1$. Which of the following statements are true?

(a) $P(A \mid B) + P(A^C \mid B) = 1$

(b) $P(A \mid B) + P(A \mid B^C) = 1$

(c) $P(A \mid B) + P(A^C \mid B^C) = 1$

[3] Consider the two events A and B such that $P(A) = 1/4$, $P(B \mid A) = 1/2$ and $P(A \mid B) = 1/4$. Which of the following statements are true?

(a) A and B are mutually exclusive events,

(b) $A \subset B$,

(c) $P(A^C \mid B^C) = 3/4$,

(d) $P(A \mid B) + P(A \mid B^C) = 1$

[4] Consider an urn in which 4 balls have been placed by the following scheme. A fair coin is tossed, if the coin comes up heads, a white ball is placed in the urn otherwise a red ball is placed in the urn.

(a) What is the probability that the urn will contain exactly 3 white balls?

(b) What is the probability that the urn will contain exactly 3 white balls, given that the first ball placed in the urn was white?

[5] A random experiment has three possible outcomes, A, B and C, with probabilities p_A, p_B and p_C. What is the probability that, in independent performances of the experiment, A will occur before B?

[6] A system composed of n separate components is said to be a parallel system if it functions when at least one of the components functions. For such a system, if component i, independent of other components, functions with probability p_i, $i=1\ (1)\ n$, what is the probability that the system functions?

[7] A student has to sit for an examination consisting of 3 questions selected randomly from a list of 100 questions. To pass, the student needs to answer correctly all the three questions. What is the probability that the student will pass the examination if he knows the answers to 90 questions on the list?

[8] A person has three coins in his pocket, two fair coins (heads and tails are equally likely) but the third one is biased with probability of heads $2/3$. One coin selected at random drops on the floor, landing heads up. How likely is it that it is one of the fair coins?

[9] A slip of paper is given to A, who marks it with either a '$+$' or a '$-$' sign, with a probability $1/3$ of writing a '$+$' sign. A passes the slip to B, who may either leave it unchanged or change the sign before passing it to C. C in turn passes the slip to D after perhaps changing the sign; finally D passes it to a referee after perhaps changing the sign. It is further known that B, C and D each change the sign with probability $2/3$. Find the probability that A originally wrote a '$+$' given that the referee sees a '$+$' sign on the slip.

[10] Each of the three boxes A, B and C, identical in appearance, has two drawers. Box A contains a gold coin in each drawer, box B contains a silver coin in each drawer and box C contains a gold coin in one drawer and silver coin in the other. A box is chosen at random and one of its drawers is then chosen at random and opened, and a gold coin is found. What is the probability that the other drawer of this box contains a silver coin?

[11] Each of four persons fires one shot at a target. Let C_k denote the event that the target is hit by person k, $k = 1, 2, 3, 4$. If the events C_1, C_2, C_3, C_4 are independent and if $P(C_1) = P(C_2) = 0.7$, $P(C_3) = 0.9$ and $P(C_4) = 0.4$, compute the probability that: (a) all of them hit the target; (b) no one hits the target; (c) exactly one hits the target; (d) at least one hits the target.

[12] Let $A_1, A_2, \ldots A_n$ be n independent events. Show that

$$P\left(\bigcap_{i=1}^{n} A_i^C\right) \le \exp\left(-\sum_{i=1}^{n} P(A_i)\right)$$

[13] Give a counter example to show that pairwise independence of a set of events $A_1, A_2, \ldots A_n$ does not imply mutual independence.

[14] Let A and B be two events. Assume $P(A) > 0$ and $P(B) > 0$. Prove that

(a) if A and B are mutually exclusive $(A \cap B = \phi)$ then A and B are not independent; and

(b) if A and B are independent then A and B are not mutually exclusive.

[15] We say that B carries negative information about event A if $P(A|B) < P(A)$. Let A, B and C be three events such that B carries negative information about A and C carries negative information about B. Is it true that C carries negative information about A? Prove your assertion.

[16] Let A and B be two independent events such that $P(A \cap B) = \frac{1}{6}$.

(a) If $P(\textit{neither } A \textit{ nor } B \textit{ occurs}) = \frac{1}{3}$, find $P(A)$ and $P(B)$.

(b) If $P(A \textit{ occurs and } B \textit{ does not occur}) = \frac{1}{3}$, find $P(A)$ and $P(B)$.

Are $P(A)$ and $P(B)$ uniquely defined in the above cases?

[17] Let A, B and C be independent events. In terms of $P(A)$, $P(B)$ and $P(C)$, express for $k = 0, 1, 2, 3,$

(a) $P(\textit{exactly } k \textit{ of the events } A, B, C \textit{ occur})$,

(b) $P(\textit{at least } k \textit{ of the events } A, B, C \textit{ occur})$ and

(c) $P(\textit{at most } k \textit{ of the events } A, B, C \textit{ occur})$.

[18] An urn contains M balls of which M_w are white.

(a) Let a random sample of size n be drawn from the urn either with replacement or without replacement. For $j = 1,\ldots,n$ let B_j be the event that the ball drawn on j^{th} draw is white. For $k = 1, \ldots, n,$ let A_k be the event that the sample contains exactly k white balls. Find $P(B_j | A_k)$ for $j, k = 1,\ldots,n$.

(b) Let n balls be drawn from the urn and laid aside (not replaced in the urn), their colour not noted. If another ball is now drawn, what is the probability that it will be white?

[19] A die is loaded in such a way that probability of a given number turning up is proportional to that number. What is the probability of rolling an even number, given that a number less than 5 turns up?

[20] A male rat is either doubly dominant (AA) or heterozygous (Aa), owing to Mendelian law, the probability of either being true is $\frac{1}{2}$. The male rate is bred to a doubly recessive (aa) female rat. If the male is doubly dominant, the offspring will exhibit dominant characteristics; and if heterozygous, the offspring will exhibit the dominant characteristics half of the times and recessive characteristics half of the times. Suppose all of three offsprings exhibit dominant characteristics, what is the probability that the male is doubly dominant?