QUIZ 1, MTH309A
TOTAL MARKS: 5

ROLL NO:
NAME:

Instructions:

(1) Tick (✓) ALL correct answers among the options given. Illegible answers will be taken as incorrect.
(2) You get no credit for rough work. No extra pages will be supplied.
(3) $\mathbb{R}$ and $\mathbb{C}$ denote the set of real numbers and the set of complex numbers respectively.
(4) You may refer to your own class notes. Searching in books/internet is not allowed.

Problems:

Q1. The statement 'The set of Rational numbers is a Borel subset of $\mathbb{R}$' is [1]
(a) true.
(b) false.

Q2. Let μ be a probability measure on $(\Omega, \mathcal{F})$. Then the statement '$\mu(A \setminus B) = \mu(A) - \mu(B)$ for all $A, B \in \mathcal{F}$ with $B \subseteq A$' is [$\frac{1}{2}$]
(a) true.
(b) false.

Q3. Let A and B be two sets with probability 1 in a probability space $(\Omega, \mathcal{F}, \mathbb{P})$. Then the statement '$\mathbb{P}(A \cap B) = \mathbb{P}(A)\ \mathbb{P}(B)$' [1]
(a) is true.
(b) is false.
(c) can not be determined from the given hypothesis.

Q4. Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a probability space. Consider the collection $\mathcal{A} := \{A \in \mathcal{F} \mid \mathbb{P}(A) = 0 \text{ or } 1\}$. Then the fact '$\mathcal{A}$ is a σ-field' [1]
(a) is true.
(b) is false.
(c) can not be determined from the given hypothesis.

Q5. Let $\mathbb{P}_1$ and $\mathbb{P}_2$ be two probability measures on a measurable space $(\Omega, \mathcal{F})$. Consider the collection $\mathcal{C} := \{A \in \mathcal{F} \mid \mathbb{P}_1(A) = \mathbb{P}_2(A)\}$. Then $\mathcal{C}$ is [$\frac{1}{2}$]
(a) non-empty.
(b) closed under complementation.
(c) a Monotone class.

Q6. Fix $t \in \mathbb{R}$. Consider the following functions: $f, g : \mathbb{R} \to \mathbb{R}$ and $h : \mathbb{R} \to \mathbb{C}$ defined by

$$f(x) := \sin(tx),\ g(x) := \cos(tx),\ h(x) := e^{itx}.$$

Then [$\frac{1}{2}$]
(a) Only f and g is Borel measurable, h is not.
(b) All are Borel measurable.
(c) None are Borel measurable.

Q7. Consider the function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) := \exp(-\frac{x^2}{2}), x \in \mathbb{R}$. Then the statement 'The function f can be uniformly approximated by simple functions on $(\mathbb{R}, \mathbb{B}_{\mathbb{R}})$' is [$\frac{1}{2}$]
(a) true.
(b) false.

Date: January 28, 2019. Time: 14:00 - 14:50 hrs.