QUIZ 3, MTH309A
TOTAL MARKS: 5

ROLL NO:

NAME:

Instructions:

(1) Tick (✓) ALL correct answers among the options given. Illegible answers will be taken as incorrect. You get no credit for rough work. No extra pages will be supplied.

(2) You may refer to your own class notes. Searching in books/internet is not allowed.

Problems:

Q1. Let $\{X_n\}$ be a sequence of real valued, independent random variables on a probability space $(\Omega, \mathcal{F}, \mathbb{P})$ with $X_n \sim N(\frac{1}{n^2}, \frac{1}{n^2}), \forall n$. Then [1]

(a) $\sum_n X_n$ converges almost surely.

(b) $\sum_n X_n$ converges only on a set A with $0 < \mathbb{P}(A) < 1$.

(c) $\sum_n X_n$ diverges almost surely.

(d) none of the above statements can be determined from the given information.

Q2. Let X and Y be real valued random variables defined on a probability space $(\Omega, \mathcal{F}, \mathbb{P})$. Then X and Y have the same distribution if [1]

(a) they have the same distribution function.

(b) $\mathbb{P}(|X - Y| > \epsilon) = 0$ for some $\epsilon > 0$.

(c) $\mathbb{E}X$ and $\mathbb{E}Y$ exist and they are equal.

(d) they have the same characteristic function.

Q3. Let Φ be the characteristic function of a random variable X. Identify which of the following are also characteristic functions of some other random variables. [1]

(a) $\Phi + 1$.

(b) $\bar{\Phi}$, the complex conjugate of Φ.

(c) $|\Phi|^2$.

(d) None of the above.

Q4. Let $\{X_n\}$ be an i.i.d sequence of random variables on the same probability space and with distribution $Poisson(3)$. Let $S_n := X_1 + \cdots + X_n, \forall n \geq 1$. Then [1]

(a) $\frac{S_n}{n} \xrightarrow{n\to\infty} 3$ in probability.

(b) $\frac{S_n}{n} \xrightarrow{n\to\infty} 9$ in probability.

(c) $\frac{S_n}{n} \xrightarrow{n\to\infty} 3$ a.s..

(d) $\frac{S_n}{n} \xrightarrow{n\to\infty} 9$ a.s..

Please turn over.

Date: April 6, 2019. Time: 14:00 - 14:50 hrs.

Q5. Let μ denote the Lebesgue measure on $(\mathbb{R}^2, \mathbb{B}_{\mathbb{R}^2})$. Let X and Y be two real valued, independent, standard Gaussian random variables defined on some probability space. Let ν denote the joint law of $\binom{X}{Y}$. Then [1]

(a) ν is not absolutely continuous with respect to μ.

(b) ν is absolutely continuous with respect to μ and with the Radon-Nikodym density given by

$$\frac{d\nu}{d\mu}(x,y) = \frac{1}{2\pi} exp\left(-\frac{x^2+y^2}{2}\right), \forall x, y \in \mathbb{R}.$$

(c) $\nu \ll \delta_{\binom{0}{0}}$.

(d) None of the above.