MTH309A PRACTICE PROBLEMS SET 2

AshBk = Probability & Measure Theory (2nd Edition), Robert B. Ash and Catherine A. Doléans-Dade. Elsevier.

ChungBk = A Course in Probability Theory (3rd Edition), Kai Lai Chung. Academic Press (Elsevier).

Question 1. AshBk [pp. 2-3] Problems 2-6,

Question 2. ChungBk [pp. 19-21] Exercises 4, 12 [Terminology: B.F. = Borel Field = σ-field], [pp. 24-25] 1, 2, 7, 8, [pp. 35-36] Theorem 3.1.2

Question 3. Let $\{A_n\}$ be a sequence of subsets of Ω. For all $\omega \in \Omega$, prove the following statements.

$$\liminf_{n\to\infty} 1_{A_n}(\omega) = 1_{\liminf_{n\to\infty} A_n}(\omega),$$

and

$$\limsup_{n\to\infty} 1_{A_n}(\omega) = 1_{\limsup_{n\to\infty} A_n}(\omega).$$

Question 4. Let $(\Omega_i, \mathcal{F}_i), i = 1, 2$ be measurable spaces and let $\mathcal{A}$ be a collection of subsets of Ω_2 which generates $\mathcal{F}_2$. Assume that $f : \Omega_1 \to \Omega_2$ satisfies $f^{-1}(A) \in \mathcal{F}_1, \forall A \in \mathcal{A}$. Show that f is measurable.

Question 5. Show that all continuous maps $f : \mathbb{R}^n \to \mathbb{R}^m$ are Borel measurable. (Hint: We have discussed the case $f : \mathbb{R} \to \mathbb{R}$. Generalize the argument.)

Question 6. Let $\{A_n\}$ be a sequence of events in $(\Omega, \mathcal{F}, \mathbb{P})$. Prove Boole's inequality, i.e.

$$\mathbb{P}\left(\bigcup_n A_n\right) \leq \sum_n \mathbb{P}(A_n).$$

Question 7. Let f and g be $\mathbb{R}$ valued Borel measurable functions on $(\Omega, \mathcal{F})$. Fix $A \in \mathcal{F}$ and define the function $h : \Omega \to \mathbb{R}$ by

$$h(\omega) = \begin{cases} f(\omega), \ \omega \in A, \\ g(\omega), \ \omega \in A^c. \end{cases}$$

Show that h is also Borel measurable.