MTH309A PRACTICE PROBLEMS SET 3

AshBk = Probability & Measure Theory (2nd Edition), Robert B. Ash and Catherine A. Doléans-Dade. Elsevier.

ChungBk = A Course in Probability Theory (3rd Edition), Kai Lai Chung. Academic Press (Elsevier).

Question 1. AshBk [p. 34] Problem 3, [p. 43] Problem 1 (2nd line)

Question 2. ChungBk [pp. 40-41] Exercises 1, 10

Question 3. Let $\{f_n\}$ be a sequence of $\mathbb{R}$ valued Borel measurable functions defined on Ω.

(a) Show that the functions $\omega \mapsto \sup_n f_n(\omega)$ and $\omega \mapsto \inf_n f_n(\omega)$ are $\bar{\mathbb{R}}$ valued Borel measurable functions.

(b) Show that the functions $\omega \mapsto \limsup_n f_n(\omega)$ and $\omega \mapsto \liminf_n f_n(\omega)$ are $\bar{\mathbb{R}}$ valued Borel measurable functions.

Question 4. Let $X, Y : \Omega \to \mathbb{R}$ be Borel measurable. Assume that Y is $\sigma(X)/\mathbb{B}_{\mathbb{R}}$ measurable. Show that there exists a Borel measurable function $f : \mathbb{R} \to \mathbb{R}$ such that $Y = f \circ X = f(X)$.
Remark: (not included in the exercise) If $X : \Omega \to \mathbb{R}$ and $f : \mathbb{R} \to \mathbb{R}$ are measurable, then so is $f \circ X = f(X)$. This is the converse.