MTH309A PRACTICE PROBLEMS SET 4

AshBk = Probability & Measure Theory (2nd Edition), Robert B. Ash and Catherine A. Doléans-Dade. Elsevier.

ChungBk = A Course in Probability Theory (3rd Edition), Kai Lai Chung. Academic Press (Elsevier).

Question 1. AshBk 1.5.9 Theorem part (d)

Question 2. ChungBk [pp. 46] Exercise 3

Question 3. Let μ and ν be two probability measures on $(\mathbb{R}, \mathbb{B}_{\mathbb{R}})$ with the same distribution function. Show that $\mu = \nu$.

Question 4. For any Borel set A in $\mathbb{R}$ and $x \in \mathbb{R}$, define $A + x := \{a + x : a \in A\}$. Show that $A + x$ is a Borel set.

Question 5. Let $\{X_n\}$ be a sequence of non-negative, integrable random variables on $(\Omega, \mathcal{F}, \mathbb{P})$. Show that

$$\mathbb{E}\left(\sum_n X_n\right) = \sum_n \mathbb{E}X_n.$$

Does the result hold if $\mathbb{E}X_n = \infty$ for some n?

Question 6. Construct a probability space $(\Omega, \mathcal{F}, \mu)$ and a real valued integrable function f on this space such that f is not bounded.