ANSWERS TO QUIZ 1, MTH309A

Multiple Choice Questions [Only one option is correct. Put a tick (✓) beside the correct option.]

Question 1. Let $\{A_n\}$ be a sequence in $\mathcal{F}$. Is the function 1_A, where $A = \liminf_{n\to\infty} A_n$, Borel measurable? [1]

✓(a) Yes. (b) No. (c) can not be determined from the given information.

Question 2. We say that a collection $\mathcal{D}$ of subsets of Ω is a π system, if it is closed under finite intersections, i.e. given $A, B \in \mathcal{D}$ implies $A \cap B \in \mathcal{D}$. Is the collection $\mathcal{D} := \{(-\infty, a] : -\infty < a < \infty\}$ of subsets of $\mathbb{R}$ a π system? [1]

✓(a) Yes. (b) No. (c) can not be determined from the given information.

Question 3. We say that a real valued set function μ defined on $\mathcal{F}$ is 'continuous from above at the empty set $\emptyset$', if for all decreasing sequence of sets $\{A_n\}$ in $\mathcal{F}$ with $A_n \downarrow \emptyset$, we have $\lim_{n\to\infty} \mu(A_n) = 0$. If μ is a σ-finite measure, is it 'continuous from above at the empty set $\emptyset$'? [1]

(a) Yes. (b) No. ✓(c) can not be determined from the given information.

Question 4. Given a probability measure μ on $(\Omega, \mathcal{F})$, exactly one of the following statements is false. Identify. [1]

(a) $|\mu(A) - \mu(B)| \leq \mu(A \setminus B) + \mu(B \setminus A)$, for all $A, B \in \mathcal{F}$.

(b) For all $A \in \mathcal{F}$, $\lim_{n\to\infty}(\mu(A))^n$ is either 0 or 1.

✓(c) $\sum_{n=1}^{\infty} \mu(A_n)$ is necessarily bounded above by 1, for all sequences $\{A_n\}$ in $\mathcal{F}$.

(d) Fix $x \in \Omega$. Then $\{A \in \mathcal{F} : \mu(A) = \delta_x(A)\}$ is a Monotone class.

P.T.O.

Date: January 29, 2020. Time: 11:00 - 11:50 hrs.

Multiple Select Questions [Number of correct option(s) can be one or more than one. Put a tick (✓) beside only the correct option(s) to get credit. No partial credits will be awarded.]

Question 5. Recall the field $\mathcal{C}$ containing finite disjoint unions of left-open right-closed intervals on $\mathbb{R}$. Suppose μ_1 and μ_2 are two measures on $\mathcal{C}$ with the property that $\mu_1((a,b]) = \mu_2((a,b]) = b - a, \forall -\infty \leq a < b \leq \infty$. Then identify the correct options. [2]

(a) μ_1 and μ_2 can be finite measures.

✓(b) μ_1 and μ_2 must be σ-finite measures.

(c) Any one of μ_1 and μ_2 can have more than one extension to $(\mathbb{R}, \mathbb{B}_{\mathbb{R}})$ and it is also possible to have no such extensions as well.

✓(d) Both μ_1 and μ_2 must extend to the same measure on $(\mathbb{R}, \mathbb{B}_{\mathbb{R}})$.

Question 6. Which of the following collections of sets generate Borel σ field on $\mathbb{R}$? Here $\mathbb{Q}$ denotes the set of rational numbers. [2]

✓(a) $(a,b), \forall a < b, a, b \in \mathbb{R} \setminus \mathbb{Q}$.

✓(b) $(-\infty, a], \forall a \in \mathbb{Q}$.

✓(c) $(a,b] \cup [c,d), \forall a < b < c < d, a, b, c, d \in \mathbb{R}$.

✓(d) all compact sets K such that there exists $a, b \in \mathbb{R}$ with $K \subseteq [a,b], a < b \leq a+1$.

Question 7. Let $f_1, f_2 : \Omega \to \mathbb{R}$ be two functions. Identify the correct options. [2]

✓(a) If f_1 and f_2 are Borel measurable, then so is $f_1 + f_2$.

(b) If f_1 and f_2 are Borel measurable, then $f_1 + f_2$ is a simple function.

✓(c) If f_1 and f_2 are simple functions, then so is $f_1 + f_2$.

✓(d) If f_1 and f_2 are simple functions, then $f_1 + f_2$ is Borel measurable.