QUIZ 1, MTH309A
TOTAL MARKS: 10

ROLL NO:
NAME:

Instructions:

(1) Illegible answers will be taken as incorrect.
(2) You get no credit for rough work. No extra pages will be supplied.
(3) You may refer to your own class notes. Searching in books/internet is not allowed. Mobiles must be switched off during the quiz.
(4) $(\Omega, \mathcal{F})$ stands for an arbitrary measurable space.

Multiple Choice Questions [Only one option is correct. Put a tick (✓) beside the correct option.]

Question 1. Let $\{A_n\}$ be a sequence in $\mathcal{F}$. Is the function 1_A, where $A = \liminf_{n\to\infty} A_n$, Borel measurable? [1]

(a) Yes. (b) No. (c) can not be determined from the given information.

Question 2. We say that a collection $\mathcal{D}$ of subsets of Ω is a π system, if it is closed under finite intersections, i.e. given $A, B \in \mathcal{D}$ implies $A \cap B \in \mathcal{D}$. Is the collection $\mathcal{D} := \{(-\infty, a] : -\infty < a < \infty\}$ of subsets of $\mathbb{R}$ a π system? [1]

(a) Yes. (b) No. (c) can not be determined from the given information.

Question 3. We say that a real valued set function μ defined on $\mathcal{F}$ is 'continuous from above at the empty set $\emptyset$', if for all decreasing sequence of sets $\{A_n\}$ in $\mathcal{F}$ with $A_n \downarrow \emptyset$, we have $\lim_{n\to\infty} \mu(A_n) = 0$. If μ is a σ-finite measure, is it 'continuous from above at the empty set $\emptyset$'? [1]

(a) Yes. (b) No. (c) can not be determined from the given information.

Question 4. Given a probability measure μ on $(\Omega, \mathcal{F})$, exactly one of the following statements is false. Identify. [1]

(a) $|\mu(A) - \mu(B)| \leq \mu(A \setminus B) + \mu(B \setminus A)$, for all $A, B \in \mathcal{F}$.
(b) For all $A \in \mathcal{F}$, $\lim_{n\to\infty}(\mu(A))^n$ is either 0 or 1.
(c) $\sum_{n=1}^{\infty} \mu(A_n)$ is necessarily bounded above by 1, for all sequences $\{A_n\}$ in $\mathcal{F}$.
(d) Fix $x \in \Omega$. Then $\{A \in \mathcal{F} : \mu(A) = \delta_x(A)\}$ is a Monotone class.

P.T.O.

Date: January 29, 2020. Time: 11:00 - 11:50 hrs.

Multiple Select Questions [Number of correct option(s) can be one or more than one. Put a tick (✓) beside only the correct option(s) to get credit. No partial credits will be awarded.]

Question 5. Recall the field $\mathcal{C}$ containing finite disjoint unions of left-open right-closed intervals on $\mathbb{R}$. Suppose μ_1 and μ_2 are two measures on $\mathcal{C}$ with the property that $\mu_1((a,b]) = \mu_2((a,b]) = b - a, \forall -\infty \leq a < b \leq \infty$. Then identify the correct options. [2]

(a) μ_1 and μ_2 can be finite measures.
(b) μ_1 and μ_2 must be σ-finite measures.
(c) Any one of μ_1 and μ_2 can have more than one extension to $(\mathbb{R}, \mathbb{B}_\mathbb{R})$ and it is also possible to have no such extensions as well.
(d) Both μ_1 and μ_2 must extend to the same measure on $(\mathbb{R}, \mathbb{B}_\mathbb{R})$.

Question 6. Which of the following collections of sets generate Borel σ field on $\mathbb{R}$? Here $\mathbb{Q}$ denotes the set of rational numbers. [2]

(a) $(a,b), \forall a < b, a, b \in \mathbb{R} \setminus \mathbb{Q}$.
(b) $(-\infty, a], \forall a \in \mathbb{Q}$.
(c) $(a,b] \cup [c,d), \forall a < b < c < d, a, b, c, d \in \mathbb{R}$.
(d) all compact sets K such that there exists $a, b \in \mathbb{R}$ with $K \subseteq [a,b], a < b \leq a + 1$.

Question 7. Let $f_1, f_2 : \Omega \to \mathbb{R}$ be two functions. Identify the correct options. [2]

(a) If f_1 and f_2 are Borel measurable, then so is $f_1 + f_2$.
(b) If f_1 and f_2 are Borel measurable, then $f_1 + f_2$ is a simple function.
(c) If f_1 and f_2 are simple functions, then so is $f_1 + f_2$.
(d) If f_1 and f_2 are simple functions, then $f_1 + f_2$ is Borel measurable.