QUIZ 1, MTH431A
TOTAL MARKS: 10

ROLL NO:
NAME:

Instructions:

(1) Illegible answers will be taken as incorrect.
(2) You get no credit for rough work. No extra pages will be supplied.
(3) You may refer to your own class notes. Searching in books/internet is not allowed. Mobiles must be switched off during the quiz.
(4) $\mathbb{Q}$ denotes the set of rational numbers.

Multiple Choice Questions [Only one option is correct. Put a tick (✓) beside the correct option.]

Question 1. Let X be a real valued random variable such that $\mathbb{E}|(X-1)^3| < \infty$. Then [1]

(1) $\mathbb{E}(X-1)^3$ and $\mathbb{E}X$ exist.
(2) $\mathbb{E}(X-1)^3$ exists, but $\mathbb{E}X$ need not exist.
(3) $\mathbb{E}(X-1)^3$ need not exist, but $\mathbb{E}X$ exists.
(4) Both of $\mathbb{E}(X-1)^3$ and $\mathbb{E}X$ need not exist.

Question 2. Let F denote the distribution function of a real valued, discrete random variable X defined on a probability space $(\Omega, \mathcal{F}, \mathbb{P})$. Then [1]

(1) The function $x \in \mathbb{R} \mapsto F(2x)$ can not be the distribution function of any random variable.
(2) $\lim_{y \uparrow x} F(y) = \lim_{y \downarrow x} F(y)$ for all $x \in \mathbb{R}$.
(3) There exist intervals such that F remains constant within these intervals.
(4) none of the above statements are true.

Question 3. Let X be a real valued, absolutely continuous random variable defined on a probability space $(\Omega, \mathcal{F}, \mathbb{P})$. Then the statement 'For any continuous function $g : \mathbb{R} \to \mathbb{R}$, the random variable $g(X)$ is continuous' [1]

(1) is true for any random variable X.
(2) is false for any random variable X.
(3) can be true or false depending on the distribution of the random variable X.

Question 4. Let X be a random variable with distribution $N(0, 2)$. Then $Var(|X|)$ is [1]

$$(1)\ 2\left(1 - \frac{4}{\pi}\right) \quad (2)\ 2 \quad (3)\ 2\left(1 - \sqrt{\frac{2}{\pi}}\right) \quad (4)\ 2\left(1 - \frac{2}{\pi}\right).$$

Date: August 30, 2019. Time: 15:00 - 15:50 hrs.

Multiple Select Questions [Number of correct option(s) is between 1 and 4. Put a tick (✓) beside only the correct option(s) to get credit. No partial credits will be awarded.]

Question 5. Identify the generators of the Borel σ-field on $\mathbb{R}$. [2]

(1) $\{(a, b] : a, b \in \mathbb{Q}\}$.
(2) $\{\{c\} : c \in \mathbb{R}\}$.
(3) $\{(a, b] \cup \{c\} : a, b, c \in \mathbb{R}\}$.
(4) $\{(0, a] : a > 0, a \in \mathbb{R})\}$.

Question 6. Let X be a real valued random variable defined on a probability space $(\Omega, \mathcal{F}, \mathbb{P})$. Which of the following information uniquely identifies the distribution of X? [2]

(1) The location(s) and size(s) of jump(s) of the distribution function.
(2) $\mathbb{P}(X \leq x^3), x \in \mathbb{R}$.
(3) $\mathbb{E}|X|$.
(4) none of the above.

Question 7. Let μ_1 and μ_2 be probability measures on a measurable space $(\Omega, \mathcal{F})$. Then which of following is/are necessarily probability measure(s)? [2]

(1) $\frac{1}{2}\mu_1 + \frac{1}{2}\mu_2$ defined by $(\frac{1}{2}\mu_1 + \frac{1}{2}\mu_2)(A) := \frac{1}{2}\mu_1(A) + \frac{1}{2}\mu_2(A), A \in \mathcal{F}$.
(2) $\frac{1}{2}\mu_1 + \frac{1}{2}$ defined by $(\frac{1}{2}\mu_1 + \frac{1}{2})(A) := \frac{1}{2}\mu_1(A) + \frac{1}{2}, A \in \mathcal{F}$.
(3) $2\mu_1 - \mu_2$ defined by $(2\mu_1 - \mu_2)(A) := 2\mu_1(A) - \mu_2(A), A \in \mathcal{F}$.
(4) $\mu_1 \times \mu_2$ defined by $(\mu_1 \times \mu_2)(A) := \mu_1(A) \times \mu_2(A), A \in \mathcal{F}$.