QUIZ 2, MTH431A, TOTAL MARKS: 10

ROLL NO:

NAME:

Instructions:

(1) You get no credit for rough work (on Multiple Choice and Multiple Select Questions). No extra pages will be supplied.

(2) You may refer to your own class notes. Searching in books/internet is not allowed. Mobiles must be switched off during the quiz.

Question 1. Let X be a real valued random variable with distribution $N(1,1)$. Compute $\mathbb{E}X^4$. [3]

Date: October 4, 2019. Time: 15:00 - 15:50 hrs.

Question 2. Let X and Y be two real valued random variables defined on the same probability space $(\Omega, \mathcal{F}, \mathbb{P})$. Assume that they are independent. Identify all the correct options below (put a tick ✓). No partial credits will be awarded. [2]

(a) The collection $\{X^2 + X, Y^3\}$ is necessarily independent.

(b) The collection $\{X^2 + X, -X, Y^3\}$ is necessarily independent.

(c) $\mathbb{P} \circ \binom{X}{Y}^{-1}(B_1 \times B_2) = \mathbb{P} \circ X^{-1}(B_1) \times \mathbb{P} \circ Y^{-1}(B_2)$ for all $B_1, B_2 \in \mathbb{B}_{\mathbb{R}}$.

(d) It is possible to have $\mathbb{P} \circ X^{-1} = \mathbb{P} \circ Y^{-1}$.

Question 3. Give an example of a real valued continuous random variable (or equivalently a probability distribution on $\mathbb{R}$ with density) such that its quantile of order $\frac{1}{3}$ is not unique. Justify your answer. [2]

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Question 4. Let X and Y be two independent random variables with p.d.f. $f(x) = \frac{1}{\pi}\frac{1}{1+x^2}, \forall x \in \mathbb{R}$.

(a) Find the distribution of $\frac{X-Y}{2}$ (i.e. compute the law/distribution function/pdf/pmf – whichever is applicable and convenient for you). [2]

(b) Is the distribution of $\frac{X+Y}{2}$ the same as that of $\frac{X-Y}{2}$? Put a tick (✓) 'Yes/ No' [1]