QUIZ 3, MTH431A
TOTAL MARKS: 10

ROLL NO:
NAME:

Instructions:

(1) Illegible answers will be taken as incorrect.
(2) You get no credit for rough work. No extra pages will be supplied.
(3) You may refer to your own class notes. Searching in books/internet is not allowed. Mobiles must be switched off during the quiz.

Multiple Choice Questions [Only one option is correct. Put a tick (✓) beside the correct option.]

Question 1. Let $X \sim Exponential(1)$. Then the value of $\mathbb{E}X^4$ is [1]

(1) 6 (2) 15 (3) 24 (4) 120.

Question 2. Let $\{A_n\}$ be a sequence of events in a probability space with $\sum_{n=1}^{\infty} \mathbb{P}(A_n) < \infty$. Then the value of $\mathbb{P}(\liminf_n A_n)$ [1]

(1) is 0.
(2) is $\frac{1}{2}$.
(3) is 1.
(4) can not be determined from the given information.

Question 3. Let X_1, X_2 and X_3 be indepedent Bernoulli$\left(\frac{1}{2}\right)$ random variables. Let $X_{(1)}, X_{(2)}$ and $X_{(3)}$ denote the corresponding order statistics. Then the value of $\mathbb{E}X_{(3)}$ [1]

(1) is $\frac{1}{2}$.
(2) is $\frac{3}{4}$.
(3) is $\frac{7}{8}$.
(4) can not be determined from the given information.

Question 4. Let X be a real valued random variable such that it has a continuous density function f. Then the statement 'there exists a Borel measurable function $g : \mathbb{R} \to \mathbb{R}$ such that $g(X) \sim Uniform(0,1)$' is [1]

(1) true. (2) false.

Date: November 8, 2019. Time: 15:00 - 15:50 hrs.

Multiple Select Questions [Number of correct option(s) can be 1, 2, 3 or 4. Put a tick (✓) beside only the correct option(s) to get credit. No partial credits will be awarded.]

Question 5. Suppose that the random variables X_1, X_2, X_3 and X_4 are defined on the same probability space and are exchangeable. In addition, assume that $\mathbb{E}(X_4 - 4)^4 < \infty$. [2]

(1) X_1, X_2 and X_3 are exchangeable.
(2) X_1, X_2, X_3, X_4 are mutually indepedent.
(3) The moments $\mathbb{E}(X_1 - 1), \mathbb{E}(X_2 - 2)^2$ and $\mathbb{E}(X_3 - 3)^3$ exist.
(4) $Cov(X_1, X_2)$ exists and $Cov(X_1, X_2) \geq -\frac{1}{3}Var(X_3)$.

Question 6. Let $X = (X_1, X_2, \cdots, X_n)^t$ be Gaussian random vector with mean vector $\mu = (\mu_1, \mu_2, \cdots, \mu_n)^t$ and Variance-Covariance matrix $\Sigma = (\sigma_{ij})_{n\times n}$. [2]

(1) Marginal distributions are also Gaussian and $X_i \sim N(\mu_i, \sigma_{ii})$ for all $i = 1, 2, \cdots, n$.
(2) Given scalars $a_1, a_2, \cdots, a_n \in \mathbb{R}$, the random variable $\sum_{i=1}^n a_i X_i$ is also Gaussian.
(3) The component random variables $X_1, X_2, \cdots, X_n$ are mutually indepedent if and only if $\sigma_{ij} = 0, \forall i \neq j$.
(4) none of the above.

Question 7. Let X and Y be two real valued random variables defined on the same probability space. Assume that they have finite second moments. [2]

(1) $X + Y$ need not have finite second moment. Only with information from the joint distribution of $\binom{X}{Y}$, we can conclude about the finiteness of this moment.
(2) $\mathbb{E}|XY|$ need not be finite. Only with information from the joint distribution of $\binom{X}{Y}$, we can conclude about the finiteness of this moment.
(3) $\inf_{c\in\mathbb{R}} \mathbb{E}(Y - c)^2$ exists and equals $Var(Y)$.
(4) $\inf_{a,b\in\mathbb{R}} \mathbb{E}(Y - aX - b)^2$ exists and equals $Var(Y)(1-\rho^2)$, where ρ is the correlation coefficient between X and Y.