QUIZ 1, MTH614A
TOTAL MARKS: 5

ROLL NO:
NAME:

Instructions:

(1) Tick (✓) ALL correct answers among the options given. Illegible answers will be taken as incorrect.
(2) You get no credit for rough work. No extra pages will be supplied.
(3) In what follows, $\{B_t\}$ will denote a 1 dimensional standard Brownian motion on some probability space $(\Omega, \mathcal{F}, \mathbb{P})$.

Problems:

Q1. The value of $\mathbb{E}(B_t^2 - B_s^2)$ for any time points s, t is [1]
- (a) 0.
- (b) $\frac{1}{2}(t-s)^2$.
- (c) $t - s$.
- (d) $\max\{s, t\}$.

Q2. Which of the following are also Brownian motions? [1]
- (a) $\{2B_{\frac{t}{4}}\}$
- (b) $\{2B_{4t}\}$
- (c) $\{X_t\}$ where $X_t = \frac{t}{2} B_{\frac{4}{t}}, t > 0$ and $X_0 = 0$.

Q3. If τ is a stopping time, then which of the following are also stopping times? [1]
- (a) $\tau + 1$.
- (b) $\max\{\tau, 2\}$.
- (c) 3τ.

Q4. The law $\mathbb{P}$ of $\{B_t\}$ (i.e. the Wiener measure) is a measure on the space of real valued functions on $[0, \infty)$ such that [$\frac{1}{2}$]
- (a) $\mathbb{P}(\{f : [0, \infty) \to \mathbb{R} \mid f \text{ is continuous}\}) = 1$.
- (b) $\mathbb{P}(\{f : [0, \infty) \to \mathbb{R} \mid f \text{ is of FV on } [0, T], \forall T > 0\}) = 1$.
- (c) $\mathbb{P}(\{f : [0, \infty) \to \mathbb{R} \mid f \text{ is differentiable}\}) = 1$.
- (d) None of the above.

Q5. Let $(\mathcal{F}_t)$ denote the natural filtration of $\{B_t\}$. Fix $s < t$. Value of $\mathbb{E}[B_t^3 | \mathcal{F}_s]$ is [1]
- (a) $\stackrel{a.s.}{=} 0$.
- (b) $\stackrel{a.s.}{=} (t-s)^3 + B_s^3$.
- (c) $\stackrel{a.s.}{=} (t-s)^3 + 3(t-s)^2 B_s + 3(t-s) B_s^2 + B_s^3$
- (d) $\stackrel{a.s.}{=} 3(t-s) B_s + B_s^3$

Q6. Let $\{X_t\}$ be a 2 dimensional standard Brownian motion. Let $\{X_t^1\}$ and $\{X_t^2\}$ denote the 1 dimensional co-ordinate processes. Then [$\frac{1}{2}$]
- (a) $\{X_t^1\}$ and $\{X_t^2\}$ are independent.
- (b) $\{X_t^1\}$ and $\{X_t^2\}$ are one dimensional standard Brownian motions.
- (c) Only one of $\{X_t^1\}$ and $\{X_t^2\}$ is a standard Brownian motion and the other is not.

Date: February 14, 2019.