QUIZ 2, MTH614A
TOTAL MARKS: 5

ROLL NO:

NAME:

Instructions:

(1) You get no credit for rough work. No extra pages will be supplied. You may use your class notes.

(2) In what follows, $\{B_t\}$ will denote a 1 dimensional standard Brownian motion on some probability space $(\Omega, \mathcal{F}, \mathbb{P})$.

Problems:

Q1. Tick (✓) ALL correct answers among the options given. [1]

(a) $\{\int_0^t B_s ds\}$ is a process with continuous paths.

(b) $\{\int_0^t B_s ds\}$ is a process with FV paths.

(c) $\{\int_0^t B_s ds\}$ is a process with differentiable paths.

(d) $\int_0^1 B_s ds$ is a Gaussian random variable.

Q2. Tick (✓) ALL correct answers among the options given. Let $V \in \mathcal{L}(B, [0,T])$ and let $\{V^n\}$ be a sequence of bounded, elementary processes converging to V in $\mathcal{L}^2(Leb \otimes \mathbb{P})$. Then $\lim_{n\to\infty} \int_0^T V_t^n \, dB_t = \int_0^T V_t \, dB_t$ is true in the sense of [1]

(a) convergence in $\mathcal{L}^2(\mathbb{P})$.

(b) convergence in $\mathcal{L}^p(\mathbb{P})$ for all $p \in [1,2]$.

(c) convergence in probability.

(d) almost sure convergence.

Q3. Let $f : \mathbb{R} \to \mathbb{R}$ be a C^2 function. Then the covariance between $\int_0^1 f(B_s)\, dB_s$ and $\int_0^2 f(B_s)\, dB_s$ is ________________________. [1]

Q4. Write down the mean and variance of the random variable $\int_0^T s\, dB_s$ __________________, ________________________. $[\frac{1}{2} + \frac{1}{2}]$

Q5. Using Itô formula, find the SDE satisfied by $\{\exp(B_t - \frac{1}{2}t)\}$ [1]

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Date: March 14, 2019.