QUIZ 3, MTH614A
TOTAL MARKS: 5

ROLL NO:

NAME:

Instructions:

(1) You get no credit for rough work. No extra pages will be supplied. You may use your class notes.

(2) In what follows, $\{B_t\}$ will denote a 1 dimensional standard Brownian motion on some probability space $(\Omega, \mathcal{F}, \mathbb{P})$.

Problems:

Q1. Tick (✓) ALL correct answers among the options given. Let $\{X_t^x\}$ denote the unique strong solution to the equation

$$dX_t = \sigma(X_t)dB_t + b(X_t)\,dt; X_0 = x$$

where $\sigma, b : \mathbb{R} \to \mathbb{R}$ are Lipschitz continuous functions and $x \in \mathbb{R}$. Then $\lim_{n\to\infty} X_1^{\frac{1}{n}} = X_1^0$ holds [1]

(a) in probability.

(b) a.s.

(c) in $\mathcal{L}^2(\mathbb{P})$.

Q2. Write down the Quadratic Variation (co-variation) process of $\{\int_0^t B_s\,dB_s\}$

______________________________. [0.5]

Q3. Write down the unique strong solution to the equation $dX_t = dB_t - X_t\,dt; X_0 = 1$. Also find $\mathbb{E}X_t$ and $Var(X_t)$.

_________________________, _______________, _______________ [1.5 + 0.5 + 0.5]

Q4. Let $\{X_t\}$ be the unique strong solution to the equation

$$dX_t = dB_t - 2(X_t + 1)\,dt; X_0 = 1.$$

Find an equivalent probability measure Q such that

$$dX_t = dB_t^* - 2X_t\,dt; X_0 = 1,$$

where $\{B_t^*\}$ is a Brownian motion under Q. For $A \in \mathcal{F}_t, Q(A) =$ _________________________. [1]

Date: April 16, 2019.