QUIZ 1, MTH754A
TOTAL MARKS: 3

ROLL NO:
NAME:

Instructions:

(1) You have 10 mins.
(2) Tick (✓) all correct answers among the options given. Illegible answers will be taken as incorrect.
(3) Each question carries a $\frac{1}{2}$ mark.
(4) Do all rough work at the back of this sheet.

Problems:

Q1. Identify the generators of the Borel σ-field on $\mathbb{R}$.
  (a) $\{\{c\} : c \in \mathbb{Q}\}$.
  (b) $\{(a, b] : a, b \in \mathbb{Q}\}$.
  (c) $\{(a, b] \cup \{c\} : a, b, c \in \mathbb{Q}\}$.

Q2. Let μ be a finite measure on $(\Omega, \mathcal{F})$. Then the statement '$\mu(A \setminus B) = \mu(A) - \mu(B)$ for all $A, B \in \mathcal{F}$ with $B \subseteq A$' is
  (a) true.
  (b) false.

Q3. Let A, B be two sets with probability 1 in a probability space $(\Omega, \mathcal{F}, \mathbb{P})$. Then the statement '$\mathbb{P}(A \cap B) = \mathbb{P}(A)\ \mathbb{P}(B)$' is
  (a) true.
  (b) false.

Q4. The statement 'The counting measure on the set of positive integers is a σ-finite measure' is
  (a) true.
  (b) false.

Q5. Fix $t \in \mathbb{R}$. Consider the following functions: $f, g : \mathbb{R} \to \mathbb{R}$ and $h : \mathbb{R} \to \mathbb{C}$ defined by

$$f(x) := \sin(tx),\ g(x) := \cos(tx),\ h(x) := e^{itx}.$$

  Then
  (a) Only f and g is Borel measurable, h is not.
  (b) All are Borel measurable.
  (c) None are Borel measurable.

Q6. Consider the function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = [x], x \in \mathbb{R}$, where $[x]$ represents the largest integer less or equal to x. Then
  (a) f is integrable with respect to the Lebesgue measure.
  (b) f is quasi-integrable with respect to the Lebesgue measure.
  (c) $\int_{\mathbb{R}} f(x)\, dx$ does not exist.

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