QUIZ 2, MTH754A
TOTAL MARKS: 3

ROLL NO:
NAME:

Instructions:

(1) You have 10 mins.
(2) Tick (✓) all correct answers among the options given. Illegible answers will be taken as incorrect.
(3) Each question carries a $\frac{1}{2}$ mark.
(4) Do all rough work at the back of this sheet.

Problems:

Q1. Let $\mathcal{F}_1$ and $\mathcal{F}_2$ be two σ-fields on a non-empty set Ω such that $\mathcal{F}_1 \subsetneq \mathcal{F}_2$. Then
  (a) $\{\mathcal{F}_1/\mathbb{B}_\mathbb{R}$ measurable functions $\} \supsetneq \{\mathcal{F}_2/\mathbb{B}_\mathbb{R}$ measurable functions $\}$
  (b) $\{\mathcal{F}_1/\mathbb{B}_\mathbb{R}$ measurable functions $\} = \{\mathcal{F}_2/\mathbb{B}_\mathbb{R}$ measurable functions $\}$
  (c) $\{\mathcal{F}_1/\mathbb{B}_\mathbb{R}$ measurable functions $\} \subsetneq \{\mathcal{F}_2/\mathbb{B}_\mathbb{R}$ measurable functions $\}$

Q2. Let $F : \mathbb{R} \to [0,1]$ be a distribution function corresponding to a probability measure $\mathbb{P}$ on $(\mathbb{R}, \mathbb{B}_\mathbb{R})$. Then the set of discontinuities of F, i.e. $\{x \in \mathbb{R} : F(x) = F(x-)\}$ is necessarily
  (a) an empty set.
  (b) a finite or countably infinite set.
  (c) a uncountable set.

Q3. Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a probability space. Fix $p, q \in (1, \infty)$ with $p < q$. Then
  (a) $\mathcal{L}^p(\mathbb{P}) \subseteq \mathcal{L}^q(\mathbb{P})$.
  (b) $\mathcal{L}^p(\mathbb{P}) \supseteq \mathcal{L}^q(\mathbb{P})$.
  (c) $\mathcal{L}^p(\mathbb{P}) = \mathcal{L}^q(\mathbb{P})$.

Q4. Let $\mathbb{P}$ be a probability measure on $(\mathbb{R}, \mathbb{B}_\mathbb{R})$. The function $x \in \mathbb{R} \mapsto e^{itx} \in \mathbb{C}$ is in $\mathcal{L}^p(\mathbb{P}; \mathbb{C})$
  (a) for $p = 1$.
  (b) for $p \in [1, \infty)$.
  (c) for no values of $p \in \mathbb{R}$.

Q5. Let $A := [0,1] \cap \mathbb{Q}$, where $\mathbb{Q}$ denotes the set of rationals in $\mathbb{R}$. Consider the function $f : [0,1] \to \mathbb{R}$ defined as $f(x) := 1_A(x)$. Then f
  (a) is Riemann integrable as well as Lebesgue integrable.
  (b) is Riemann integrable, but not Lebesgue integrable.
  (c) is Lebesgue integrable, but not Riemann integrable.
  (d) is neither Riemann integrable, nor Lebesgue integrable.

Q6. Consider the following statement: 'Given a Lebesgue-Stieltjes measure μ on $(\mathbb{R}, \mathbb{B}_\mathbb{R})$, there exists a unique distribution function $F : \mathbb{R} \to \mathbb{R}$ such that $\mu((a,b]) = F(b) - F(a)$.' This statement is
  (a) true.
  (b) false.