QUIZ 3, MTH754A
TOTAL MARKS: 3

ROLL NO:
NAME:

Instructions:

(1) You have 10 mins.
(2) Tick (✓) all correct answers among the options given. Illegible answers will be taken as incorrect.
(3) Each question carries a $\frac{1}{2}$ mark.
(4) Do all rough work at the back of this sheet.

Problems:

Q1. Let F be the distribution function of a real valued random variable. Fix $a, b \in \mathbb{R}$ with $a < b$. The statement 'The function $x \in [a, b] \to F(x)$ is Riemann integrable' is
  (a) true.
  (b) false.

Q2. Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a probability space. Fix $p \in [1, \infty)$. Convergence in $\mathcal{L}^p(\mathbb{P})$ implies convergence in $\mathcal{L}^q(\mathbb{P})$ for all
  (a) $q \in [1, p]$.
  (b) $q \in [p, \infty)$.
  (c) none of the above.

Q3. Identify the type of convergences which are metrizable.
  (a) $\mathcal{L}^p(\mathbb{P})$ convergence for $p \in [1, \infty]$.
  (b) a.s. convergence.
  (c) convergence in probability.
  (d) none of the above.

Q4. Let X, Y be independent real valued random variables on a probability space $(\Omega, \mathcal{F}, \mathbb{P})$. Let $f, g : \mathbb{R} \to \mathbb{R}$ be measurable. Then the random variables $f(X), g(Y)$ are
  (a) independent.
  (b) not necessarily independent.

Q5. Let $\{\mathcal{F}_i\}_{i \in \Lambda}$ be an arbitrary collection of σ-fields on a non-empty set Ω. Then the Cartesian product $\prod_{i \in \Lambda} \mathcal{F}_i = \{\prod_{i \in \Lambda} A_i : A_i \in \mathcal{F}_i\}$ is a
  (a) π-system.
  (b) σ-field.
  (c) Monotone class.

Q6. Let $\{X_n\}$ be an iid sequence of random variables defined on a probability space $(\Omega, \mathcal{F}, \mathbb{P})$. Suppose $\mathbb{E}X_1 = 0$ and $Var(X_1) = 1$. Define $X :\stackrel{d}{=} \lim_n n^{-2}(X_1 + \cdots + X_n)$ ($\stackrel{d}{=}$ denotes equality in distribution). Then
  (a) the limit may not exist and as such, X need not be defined.
  (b) the limit X exists, but can not be determined from the given information.
  (c) $X \stackrel{d}{=} N(0, 1)$.
  (d) $X = 0$.

Date: November 15, 2018.