Computational study of flow over flapping and hovering aerofoils using high order compact and filtering schemes

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Scope of the presentation

To solve unsteady aerodynamics of hovering and flapping aerofoil motion with

- Correct formulation of the problem using Ψ and ω as dependent variables
- A new method to solve Pressure Poisson Equation in truncated part of the computational domain on a non-staggered grid
- High spectral accuracy compact scheme for solving Vorticity Equation Solver
- Use of high order filters for accelerating the computation

Hovering and Flapping aerofoil motions

When an airfoil's horizontal translatory motion is combined with pitching and vertical heaving motions, flapping motion is produced. In hovering motion, the horizontal translatory motion is absent and instead of vertical heaving motion, the airfoil performs horizontal heaving motion.

Types of hovering motion

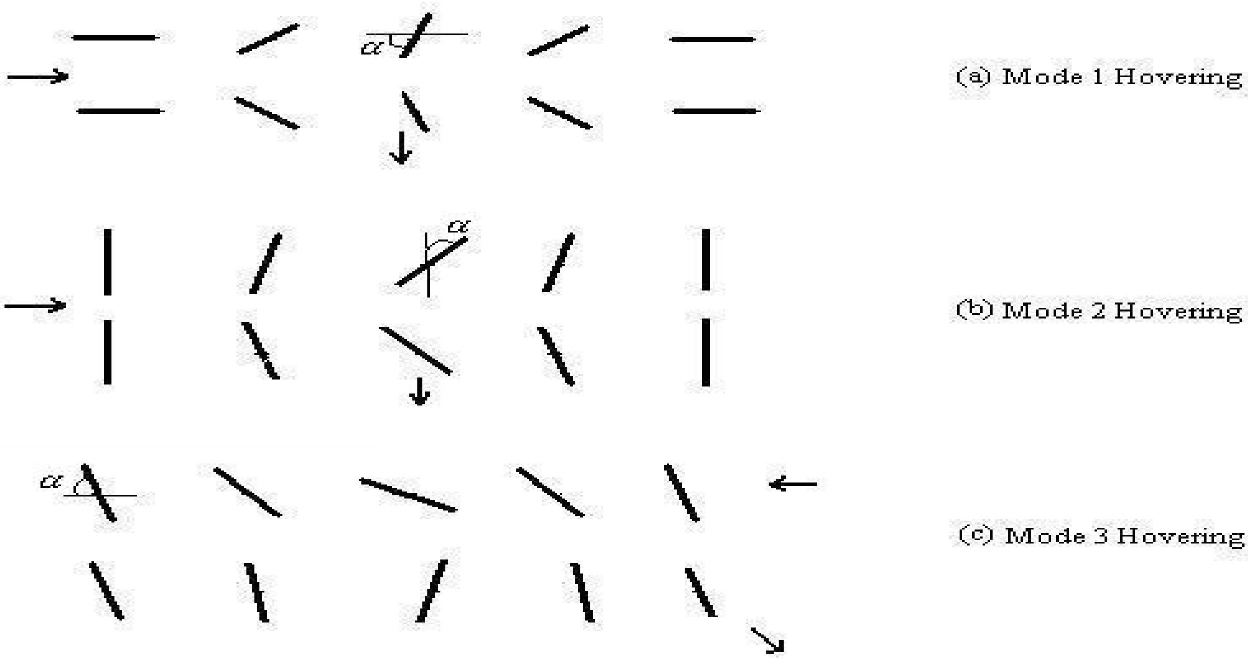
Let the translatory and angular motion for hovering airfoils be given by

$$h = h_a \sin(2\pi ft)$$
$$\alpha = \alpha_o + \alpha_a \sin(2\pi ft + \varphi)$$

respectively.

Mode 1	Mean α = 0	Φ=-90
Mode 2	Mean α = 90	Φ=90
Mode 3	0<Mean α<90	Φ~close to 90

Types of hovering motion (Contd.)



Mathematical Formulation

$VTE:$

$$h_1 h_2 \frac{\partial \omega_r}{\partial t} + h_2 u \frac{\partial \omega_r}{\partial t} + h_1 v \frac{\partial \omega_r}{\partial t} = \frac{1}{\mathrm{Re}} \left\{ \frac{\partial}{\partial \xi}\left[\frac{h_2}{h_1}\frac{\partial \omega_r}{\partial \xi}\right] + \frac{\partial}{\partial \eta}\left[\frac{h_1}{h_2}\frac{\partial \omega_r}{\partial \eta}\right]\right\} - 2\Omega h_1 h_2$$

$SFE:$

$$\frac{\partial}{\partial \xi}\left[\frac{h_2}{h_1}\frac{\partial \psi}{\partial \xi}\right] + \frac{\partial}{\partial \eta}\left[\frac{h_1}{h_2}\frac{\partial \psi}{\partial \eta}\right] = -h_1 h_2 \omega$$

$PPE:$

$$\frac{\partial}{\partial \xi}\left[\frac{h_1}{h_2}\frac{\partial P}{\partial \xi}\right] + \frac{\partial}{\partial \eta}\left[\frac{h_2}{h_1}\frac{\partial P}{\partial \eta}\right] = \frac{\partial (h_2 v \omega_I)}{\partial \xi} - \frac{\partial (h_1 u \omega_I)}{\partial \eta}$$

Mathematical Formulation(Contd.)

Non Dimensionalization is done using c and U.

- For flapping case U is the freestream velocity
- For hovering case U=fc

Solution Procedure

Flapping Case (Only vertical heaving)	NACA 0014
Hover Mode 2	NACA 0015
Combined Flapping and Hover Case	NACA 0014

Grid Generation

Orthogonal grids are generated using the hyperbolic grid generation procedure given in Nair & Sengupta[9]. 257 points are taken along the airfoil surface and 300 points are taken perpendicular to the airfoil surface.

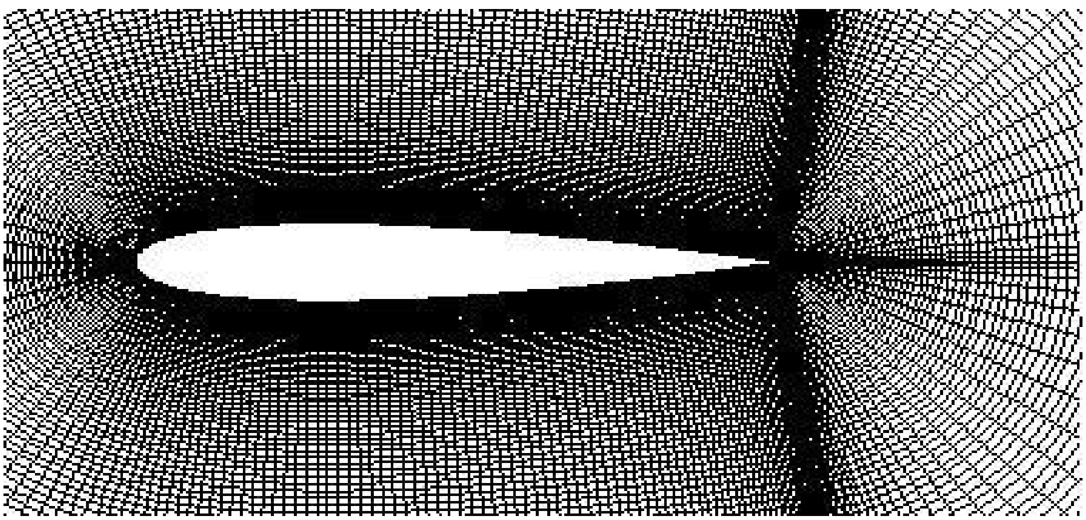
$$S(\eta) = H\left\{1 - \frac{\tanh\left[\beta(1-2\eta)\right]}{\tanh\beta}\right\}$$

$$0 \le \eta \le 0.5$$

$$0 \le S \le H$$

$$\beta = 1.55$$

Grid Generation(Contd.)



Streamfunction Solver

Two streamfunction equatios:

$$\nabla^2 \psi_I = -\omega_I$$
$$\nabla^2 \psi_r = -\omega_r$$

Instead of the second SFE, the following analogous equation is solved

$$\nabla^2 \psi_N = -\omega_I$$

Streamfunction Solver(Contd.)

where

$$\psi_N = \psi_r - \frac{\Omega}{2}(R_r^2 - R_{oI}^2) + u_{oI} y_I - v_{oI} x_I$$

Boundary Condtions:

No slip boundary condition at surface

At outer boundary :

$$\left.\frac{\partial \psi_I}{\partial \eta}\right|_{\infty} = U_{\infty} \left.\frac{\partial y_I}{\partial \eta}\right|_{\infty}$$

$$\left.\frac{\partial \psi_N}{\partial \eta}\right|_{\infty} = U_{\infty} \left.\frac{\partial y_I}{\partial \eta}\right|_{\infty}$$

Vorticity Solver

- Convective terms are evaluated using very high order compact schemes[25].

$$b_{j-1}u'_{j-1} + b_j u'_j + b_{j+1}u'_{j+1} = \frac{1}{h}\sum_{k=-2}^{2} a_{j+k}u_{j+k}$$

$$u'_1 = \frac{(-3u_1 + 4u_2 - u_3)}{2h}$$

$$u'_2 = \left[(\frac{2\beta}{3} - \frac{1}{3})u_1 - (\frac{8\beta}{3} + \frac{1}{2})u_2 + (4\beta + 1)u_3 - (\frac{8\beta}{3} + \frac{1}{6})u_4 + \frac{2\beta}{3}u_5 \right] / h$$

Vorticity Solver(Contd.)

- Explicit Fourth order disspation is added to calculate the first order derivatives.
- Diffusion terms are discretized using standard second order central differencing

Boundary Conditions:

$$\omega_r |_{\infty} = -2\Omega$$

$$\omega_r |_{body} = -\frac{1}{h_2^2}\frac{\partial^2 \psi_r}{\partial \eta^2} |_{body}$$

Vorticity Solver(Contd.)

- Four step Runge Kutta method is used for time integration.
- Eighth order filter for filtering the time integrated vorticity values

Pressure Solver

Pressure Poisson Equation :

$$\nabla^2 P = \sigma$$

Compatibility Condition:

$$\iint_{Area} \sigma dA = \oint \frac{\partial P}{\partial \eta} dl$$

LHM=RHM=0

Pressure Solver(Contd.)

$$\frac{1}{\Delta\xi^2}\{\frac{h_2}{h_1}|_{(i+1/2,j)}\ P_{(i+1,j)} + \frac{h_2}{h_1}|_{(i-1/2,j)}\ P_{(i-1,j)}\} - \{\frac{1}{\Delta\xi^2}[\frac{h_2}{h_1}|_{(i+1/2,j)}$$

$$+\frac{h_2}{h_1}|_{(i-1/2,j)]} + \frac{1}{\Delta\eta^2}[\frac{h_1}{h_2}|_{(i,j+1/2)} + \frac{h_1}{h_2}|_{(i,j-1/2)}]\}P_{(i,j)} +$$

$$\frac{1}{\Delta\eta^2}\{\frac{h_1}{h_2}|_{\{i,j+1/2)}\ P_{(i,j+1)} + \frac{h_1}{h_2}|_{(i,j-1/2)}\ P_{(i,j-1)}\}$$

$$= \frac{(h_2 v\omega)_{(i+1/2,j)} - (h_2 v\omega)_{(i-1/2,j)}}{\Delta\xi} - \frac{(h_1 u\omega)_{(i,j+1/2)} - (h_1 u\omega)_{(i,j-1/2)}}{\Delta\eta}$$

$$(h_2 v\omega)_{(i+1/2,j)} = \frac{1}{8}\{h_{2(i,j)} + h_{2(i+1,j)}\}\{v_{(i,j)} + v_{(i+1,j)}\}\{\omega_{(i,j)} + \omega_{(i+1,j)}\}$$

Pressure Solver(Contd.)

Neumann Boundary Condition:

$$\frac{h_1}{h_2}\frac{\partial P}{\partial \eta} = -h_1 u \omega_l + \frac{1}{\mathrm{Re}}\frac{\partial \omega_l}{\partial \xi} - h_1 \frac{\partial v}{\partial t}$$

$$\frac{P_{(i,2)} - P_{(i,1)}}{\Delta \eta} = \frac{1}{\mathrm{Re}}\left(\frac{h_1}{h_2}\frac{\partial \omega}{\partial \xi}\right)_{(i,3/2)} - (h_2 v \omega)_{(i,3/2)} - \left[h_1 \frac{\partial v}{\partial t}\right]_{(i,3/2)}$$

To set LHM and RHM to zero this discretized equation must be multiplied by $\frac{1}{\Delta \eta}\left[\frac{h_1}{h_2}\right]_{(i,3/2)}$

Pressure Solver(Contd.)

$$\left[h_1 \frac{\partial v}{\partial t}\right]_{(i,3/2)} = -\frac{\partial}{\partial t}\left(\frac{\partial \psi_I}{\partial \xi}\right)_{(i,3/2)}$$

PPE is solved using Conjugate Gradient Algorithm

Results

- Flapping : Re=2000, h=0.40c, k=0.4
- Hover Mode 2 : Re=27000, h=c, k=1.0, pitch amp=5 degrees
- Combined Motion: Re=20000, h=0.5c, mean alpha=5 degrees, phi=0

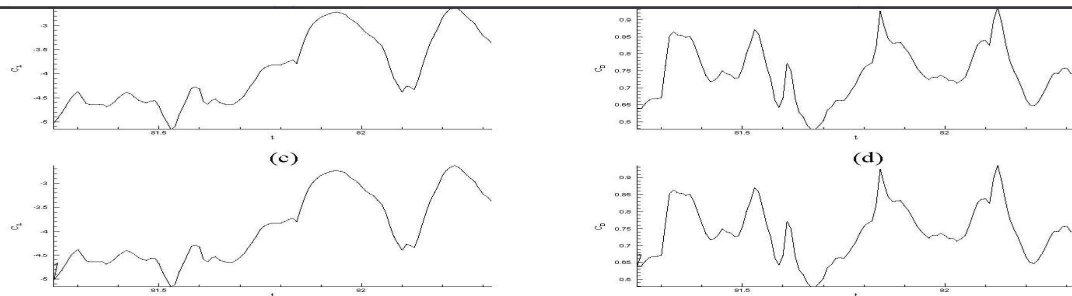


Fig.1. Solution of PPE using two different values of m. Shown are (a) C_L vs time and (b) C_D vs time for m=100 and (c) C_L vs time and (d) C_D vs time for m=150.

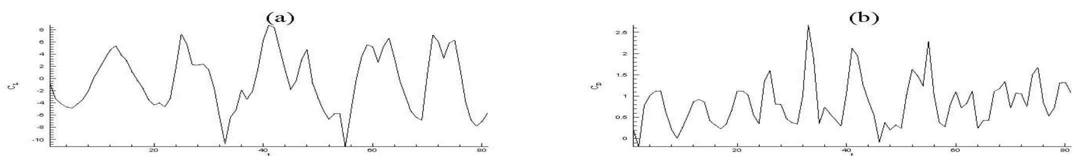


Fig.2. Loads vs time for flapping airfoil. Shown are (a) C_L vs time and (b) C_D vs time.

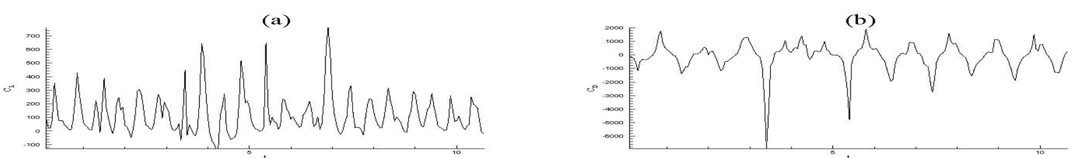


Fig.3. Loads vs time for hover mode motion. Shown are (a) C_L vs time and (b) C_D vs time.

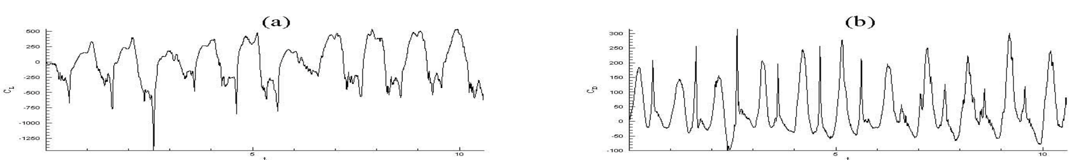


Fig.4. Loads vs time for combined flapping and hover mode motion. Shown are (a) C_L vs time and (b) C_D vs time.

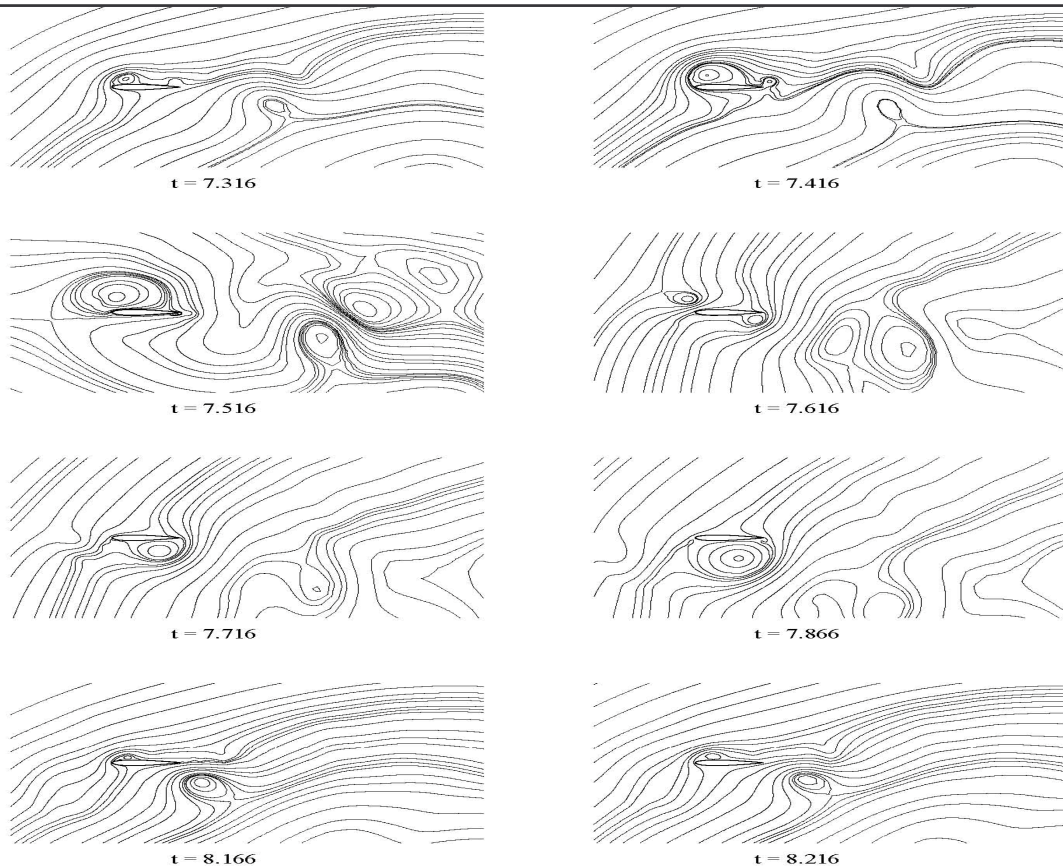


Fig.5. Streamlines in the moving body-fixed frame at indicated times for combined flapping and hover mode motion.

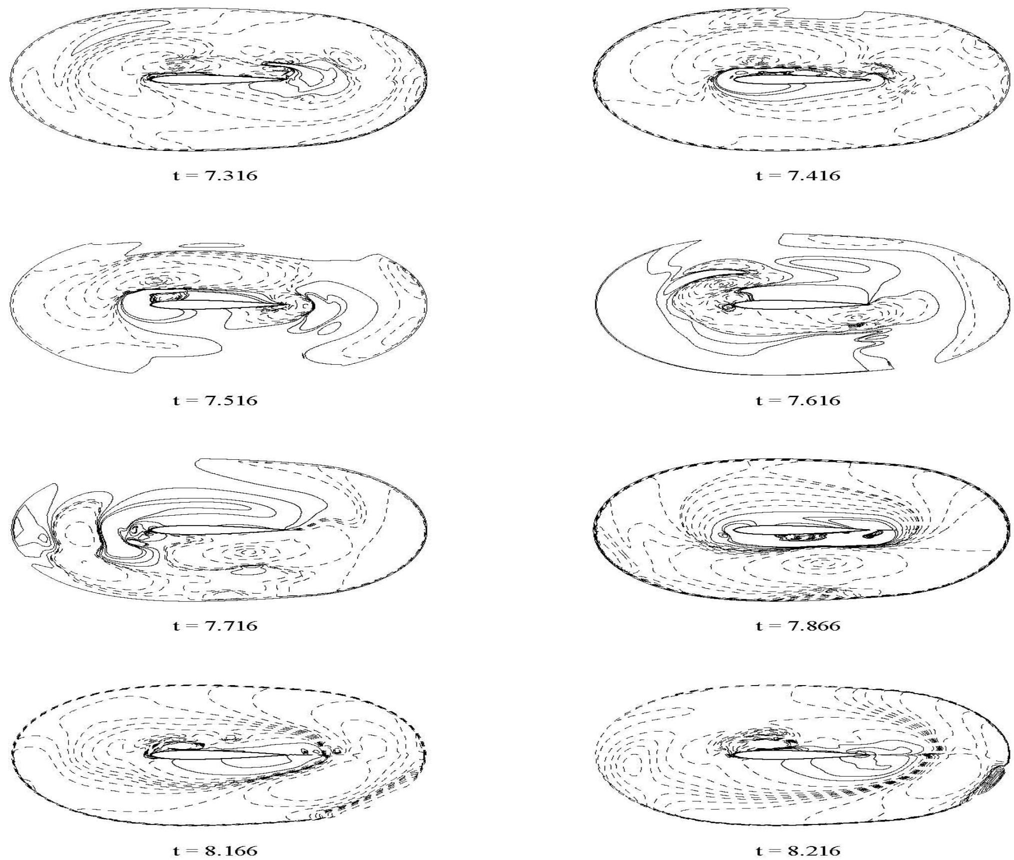


Fig. 6. Pressure contours in the moving body-fixed frame at indicated times for combined flapping and hover mode motion. Dashed lines are for negative contours.

THANK YOU