

① Velocity of camera $-\dot{x}, -\dot{y}, -\dot{z}$.

So the velocity of the object in the frame of reference of the camera is $\dot{x}, \dot{y}, \dot{z}$

Let (x, y) be the coordinate of the image of the object which is assumed to be behind the pin hole.

then,

$$x = -f \frac{X}{z}$$

$$y = -f \frac{Y}{z}$$

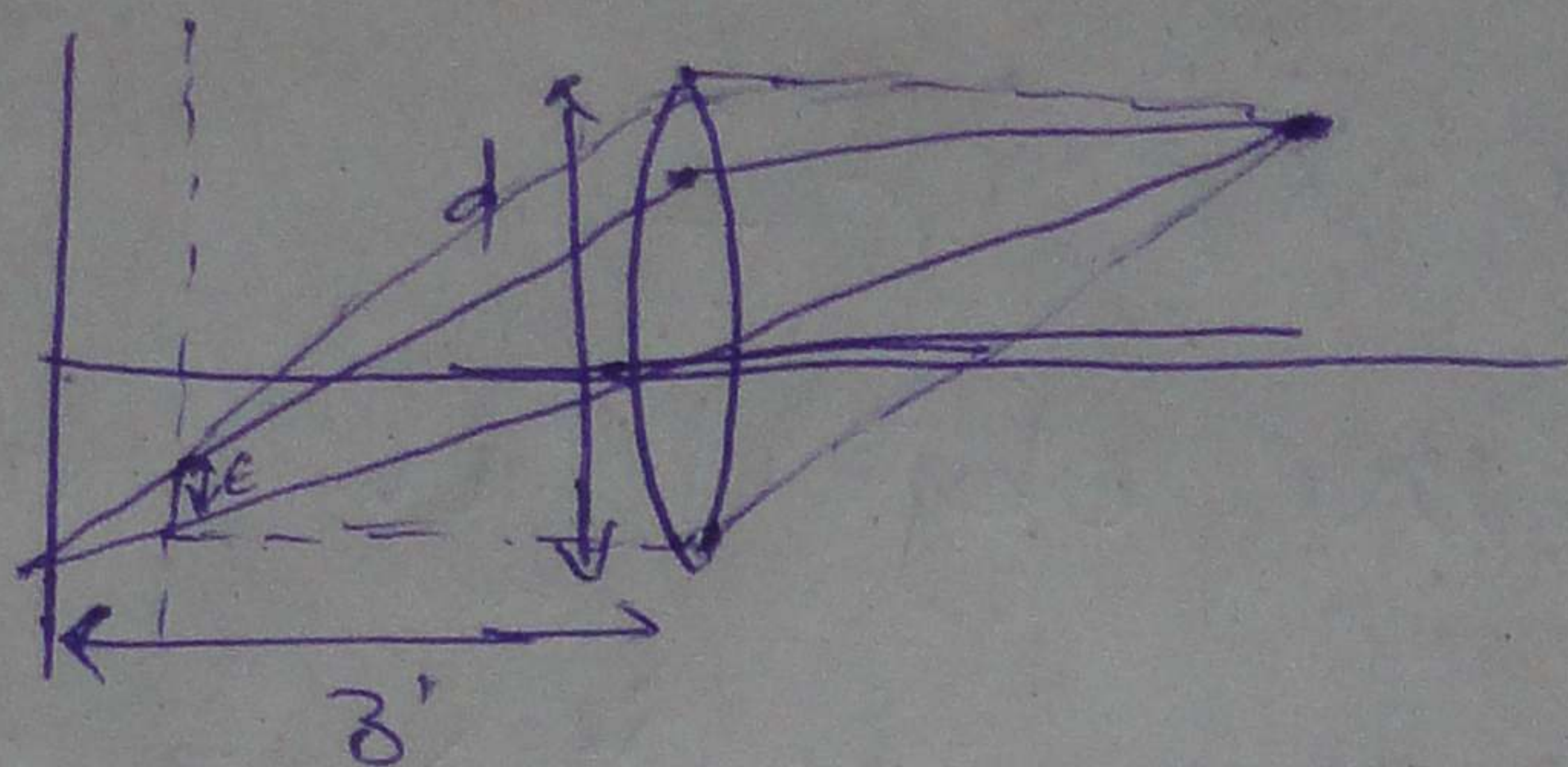
differentiating

$$\dot{x} = -f \left(\frac{z\dot{X} - X\dot{z}}{z^2} \right)$$

$$\dot{y} = -f \left(\frac{z\dot{Y} - Y\dot{z}}{z^2} \right)$$

$(\dot{x}, \dot{y})$ is the velocity of the image of the object

1(b)



By similar triangles

$$\frac{\epsilon}{d} = \frac{|z' - \hat{z}'|}{z'}$$

Without loss of generality assume

$z' > \hat{z}'$ first assume $z' > \hat{z}'$

By the equation of thin lens, the depth of a point which will focus on the image plane at $\hat{z}'$.

$$\hat{z} = f z \frac{d - \epsilon}{d + \epsilon z}$$

When $z' < \hat{z}'$

$$\hat{z} = f z \frac{d + \epsilon z}{d - \epsilon z}$$

The depth of field is the difference between the two distances

So

$$D (\text{Depth of field}) = \frac{2 \epsilon d f z (z + f)}{f^2 d^2 - \epsilon^2 z^2}$$

proved

(c) Consider a rotation about x axis
then rotation matrix R is

$$R = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix}$$

for ~~eigen vectors~~ eigenvalue

$$|R - \lambda I| = 0$$

$$\begin{vmatrix} 1-\lambda & 0 & 0 \\ 0 & \cos\theta - \lambda & -\sin\theta \\ 0 & \sin\theta & \cos\theta - \lambda \end{vmatrix} = 0$$

$$(1-\lambda) (\cos\theta - \lambda)^2 + \sin^2\theta = 0$$

$\lambda = 1$ is a solution.

In case of a general rotation, the rotation is about some vector in space and that vector remains unchanged. Also since rotation is distance preserving transform. Thus in case of general rotation there should be at least one eigenvalue equal to 1.

Physical interpretation of eigenvector

(4)

corresponding to this eigenvalue of 1.

If eigen value is 1

$$\cancel{R \cdot A =}$$

$$R \cdot v = \lambda v$$

$$R \cdot v = v \quad \{v \text{ is eigenvector}\}$$

It means the eigenvector v remains unchanged under the transformation.

Its orientation doesn't change and it

is the vector about which rotation

is taking place.

1(a) B is obtained from frame A by rotating frame A by $[R]$ and translation t .

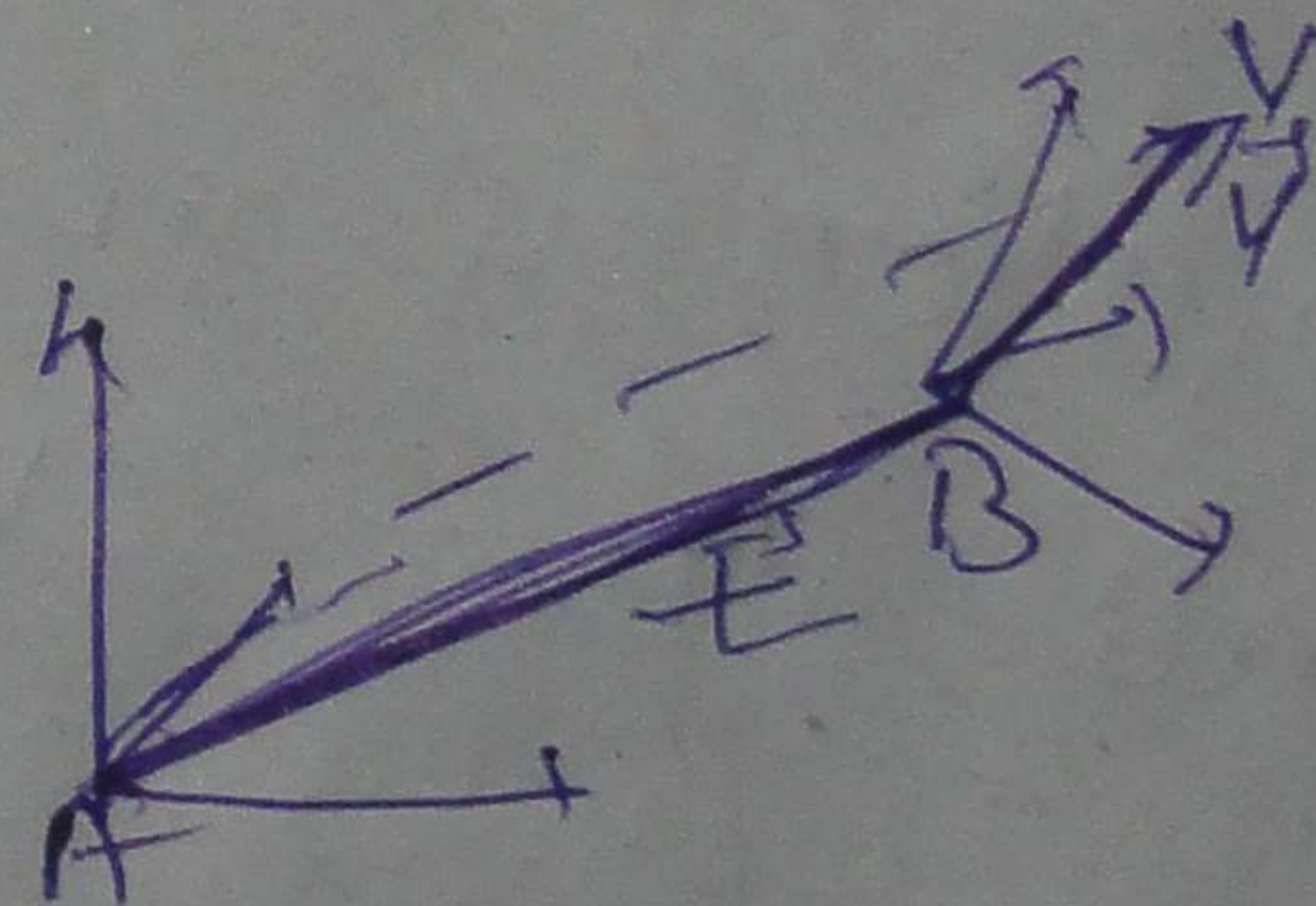
$[R]$ is rotation matrix so $[R]^T = [R]^+$

Let $\vec{v}_B$ be a vector in B.

Coordinate of frame B in A is $\vec{t}$.

$$\text{So } \vec{AV} = \vec{AB} + \vec{BV}$$

$$= \vec{t} + \underbrace{[R]^T \vec{v}_B}_{\text{to bring vector } \vec{v}_B \text{ in frame A.}}$$



10 Let the rotation be around x axis

$$R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$$

Any rigid transformation with rotation $[R]$

$$[T] = \begin{bmatrix} [R] & t \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

so for points P & Q

$$P' = [T]P \quad \& \quad Q' = [T]Q$$

$$\text{dist.}(PQ) = \|Q - P\|$$

$$\text{dist.}(P'Q') = \|Q' - P'\|$$

$$= \|\cancel{[T]}(\cancel{Q} - \cancel{P})\|$$

$$= \|[T](Q - P)\|$$

$$\text{Let } P = \begin{bmatrix} p_x \\ p_y \\ p_z \\ 1 \end{bmatrix}$$

$$Q = \begin{bmatrix} q_x \\ q_y \\ q_z \\ 1 \end{bmatrix}$$

$$\|P'Q'\| = \left\| [T] \begin{bmatrix} q_x - p_x \\ q_y - p_y \\ q_z - p_z \\ 1 \end{bmatrix} \right\|$$

$$= \left\| \begin{bmatrix} [R] & t \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} q_x - p_x \\ q_y - p_y \\ q_z - p_z \\ 1 \end{bmatrix} \right\|$$

$$q_x - p_x = \Delta x, \quad q_y - p_y = \Delta y, \quad q_z - p_z = \Delta z$$

$$\left\| [R] \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{bmatrix} \right\| = \left\| \begin{pmatrix} \Delta x \cos \theta - \Delta y \sin \theta \\ \Delta x \sin \theta + \Delta y \cos \theta \\ \Delta z \end{pmatrix} \right\|$$

$$= \sqrt{(\Delta x \cos \theta - \Delta y \sin \theta)^2 + (\Delta x \sin \theta + \Delta y \cos \theta)^2 + (\Delta z)^2}$$

$$= \sqrt{\Delta x^2 + \Delta y^2 + \Delta z^2}$$

$$= \|PQ\|$$

This will hold in case of rotation about other axes as well. Also since any general rotation can be broken down to rotation about the three axes.

This will hold for a general rotation $[R]_{\text{gen}}$ also. $([R]_{\text{gen}} = [R_x][R_y][R_z])$