

DESIGN OF UNIPOLAR (OPTICAL) ORTHOGONAL CODES AND THEIR MAXIMAL CLIQUE SETS

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CERTIFICATE

Certified that **Mr. Ram Chandra Singh Chauhan** (PhD/07/EC/539) has carried out the research work presented in this thesis entitled **“Design of Unipolar (Optical) Orthogonal Codes and Their Maximal Clique Sets”** for the award of **Doctor of Philosophy** from Uttar Pradesh Technical University, Lucknow under our supervision in the discipline of Electronics & Communication Engineering. This work is done under Teacher-Fellowship scheme of U.P.T.U. Lucknow at Electronics Engineering department of H.B.T.I. Kanpur during 2007 – 2012. The thesis embodies result of original work, and studies carried out by student himself and the contents of the thesis do not form the basis for the award of any other degree to the candidate or to anybody else from this or any other University/Institution.

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ABSTRACT

In the present age of communication, people from different regions, cultures and languages are coming closer through different mediums of communication to share their knowledge and information. These communication systems are being developed by scientists and technocrats from all across the world. Entrepreneurs, scientists and technocrats are keen to develop systems providing faster communication with higher capacity and bandwidth. They wish to build the cheapest way to provide better quality of service so that every person can afford to communicate with others in the world to share information and knowledge. This information could be anything ranging from data to audio and video.

Every day better ideas are being implemented to fulfill the basic desire of people to have better communication medium. Now-a-days, the common mediums for communication are Internet, telephone (mobile phone), television and AM/FM radio. These mediums of communication are either wired or wireless i.e. the transmitters and the receivers are connected with each other through a cable (wires) or through a wireless medium. The wireless medium may be atmosphere or tropospheric layers which reflects the radio waves with limited bandwidth (Mega-Hertz range) and power. The other mediums providing wireless communication are based on human made satellites which can provide faster communication limited up to few Mbps through stations or towers on the earth.

There is another medium which uses optical signals with huge bandwidth, of the order of Tera-Hertz, and it can provide faster communication. Optical transport can be wireless as well as wired. The optical wireless communication is done by highly directional laser beams limited by the line of sight. Optical wired medium i.e. optical fiber can provide communication to a much higher distance with higher capacity and higher speed. The optical cables between two locations have many optical fibers, thus increasing the capacity of optical channel up to thousands of Tera-Hertz. These optical channels can be shared by thousands of users at the same time without any interference. The optical channels with such huge bandwidth started attracting the researchers to explore this communication medium. A lot of hurdles have been resolved by research community since 1970 but new challenges and limits are still being faced.

Optical code division multiple access (CDMA) is a possible scheme to access the optical channel by thousands of users simultaneously with acceptable bit error rate (BER) performance. In Optical CDMA each user at one end is connected to an optical star coupler (OSC). This OSC is connected to other optical star couplers from other end through optical fibers. Each user has its own transmitter and receiver section with separate assignment of optical orthogonal codes (OOCs). The codes assigned to the transmitter section of a user (information source) will also be provided to receiver section of other user as information sink and vice versa.



These optical orthogonal codes within a set are designed by an optical orthogonal code design scheme. Since the spectral width of CDMA signal is large, this scheme is also called spread-spectrum communication.

In this thesis, the topic of deliberation is design of one dimensional as well as two dimensional unipolar (optical) orthogonal codes and their maximal clique sets by proposed general algorithms. The designed one dimensional or two dimensional unipolar (optical) orthogonal codes (OOC) are utilized for assignment of orthogonal codes to all pairs of transmitter of information source and receiver of information sink in the network. The algorithms to design one dimensional unipolar (optical) orthogonal codes and their multiple sets are being compared with already proposed schemes in the literatures for designing one dimensional orthogonal codes. An ideal scheme designing all possible sets of one dimensional unipolar orthogonal codes with maximum cardinality is assumed and compared with the proposed schemes as well as schemes in literature for relative performance evaluation. The algorithms to design two dimensional unipolar (optical) orthogonal codes and their multiple sets are being compared with already proposed schemes in the literature for designing two dimensional orthogonal codes. An ideal scheme designing all possible sets of two dimensional unipolar orthogonal codes with maximum cardinality is assumed and compared with the proposed schemes as well as schemes in literature for relative performance evaluation.

This thesis is organized into six chapters. First chapter gives the historical perspective of optical code division multiple access (OCDMA) and optical cdma codes or unipolar (optical) orthogonal codes. This historical perspective is organized into two subsections. First subsection gives the evolution of one dimensional unipolar (optical) orthogonal codes and optical cdma employing one dimensional orthogonal codes with fixed length and weight. The subsection also deals with the development of multi-length, multi-weight unipolar (optical) orthogonal codes and optical cdma employing these codes. The second subsection deals with the evolution of two dimensional unipolar (optical) orthogonal codes and optical cdma employing these codes. This subsection also gives the development of three and multi-dimensional unipolar (optical) orthogonal codes and optical cdma employing these codes. This chapter also deals with types of optical CDMA based on optical coding as well as multiple access interference with its reduction schemes. The first chapter also addresses motivation and the research problem to be resolved.

Second chapter discusses one dimensional optical orthogonal codes, their conventional representations, the conventional methods to calculate auto-correlation and cross-correlation constraints along-with the properties of sets of codes and the schemes proposed in literature finding code words. This chapter also introduces the cardinality bounds on the set of one dimensional optical orthogonal codes called Johnson's bound. The comparison of these schemes with each other and with an ideal one have also been discussed. The comparison of the scheme with the ideal one gives the idea of further improvements.



In the chapter three, the generation of one dimensional unipolar (optical) orthogonal codes in multiple sets is discussed. Each set contains the codes with maximum cardinality for given code length ' n ', given code weight ' w ', auto-correlation constraints less than or equal to λ_a , and cross-correlation constraints less than or equal to λ_c with positive integer values and boundaries like $1 \leq \lambda_a, \lambda_c < w < n$ and w is co-prime with n . The maximum cardinality or upper bound of each set of codes is given by Johnson bounds. A unique representation named by difference of positions representation (DoPR) and new simple methods for calculation of auto-correlation as well as cross-correlation constraints of one dimensional unipolar (optical) orthogonal codes are also proposed in this chapter. Two search algorithms are proposed which find multiple sets of unipolar (optical) orthogonal codes. The first algorithm finds all possible sets of unipolar (optical) orthogonal codes with maximum cardinality for code length ' n ', code weight ' w ' such that w and n are co-prime, auto-correlation constraint and cross-correlation constraint in the range lying from 1 to $w-1$ using direct search method. This algorithm works well upto $n=47$ and $w=4$ for auto-correlation and cross-correlation constraints lying from 1 to 3. The second algorithm uses clique search method to find all sets of codes not only for the same length and the same weight but also for the multi-length and multi-weight one dimensional unipolar orthogonal codes. This algorithm work well upto $n=256$ and $w=5$ for auto-correlation and cross-correlation constraints lying from 1 to 2. The algorithm work well is quoted in the sense of timing required in execution of programs.

Second algorithm is proposing the codes and their all multiple sets using clique search method which reduces computational complexity. These algorithms are generating their codes in difference of positions representation (DoPR) proposed here. These codes can be converted into proper binary sequences which can be assigned to multiple users of incoherent optical cdma system.

Fourth chapter gives details of two dimensional optical orthogonal codes used in optical CDMA systems. It describes the conventional representations and conventional methods to calculate correlation constraints. It explains the proposed schemes in literature for the design of set of two dimensional optical orthogonal codes. The Johnson's bound or cardinality for the set of two dimensional optical orthogonal codes has also been given here. The ideal scheme for design of two dimensional optical orthogonal codes has been assumed with ideal results and compared with the proposed schemes in literature. This comparison provides an idea about how close the existing schemes are to the ideal one.

Fifth chapter discusses two dimensional unipolar (optical) orthogonal codes, with a new and unique representation of two dimensional optical orthogonal codes, a novel and simple method for calculation of correlation constraints. Two new search algorithms for design of two dimensional unipolar (optical) orthogonal codes through one dimensional unipolar (optical) orthogonal codes and finding their multiple sets have been discussed. The cardinality of each code-set approach the Johnson's bound for different correlation constraints. This newly proposed scheme has also been compared with ideal one which is assumed in chapter four. The first algorithm finds all possible sets of unipolar (optical) orthogonal codes with maximum cardinality



for matrix code dimension ($L \times N$), code weight 'w' such that w and LN are co-prime, auto-correlation constraint and cross-correlation constraint from the range 1 to $w-1$ using direct search method. This algorithm works well upto $LN = 46$ and $w=4$ for auto-correlation and cross-correlation constraints lying from 1 to 3. The second algorithm uses clique search method to find all sets of codes not only for the same length and the same weight but also for the multi-length and multi-weight one dimensional unipolar orthogonal codes. This algorithm work well upto $LN= 256$ and $w=5$ for auto-correlation and cross-correlation constraints lying from 1 to 2.

Finally, in chapter six the first conclusion has drawn from the comparison of proposed one dimensional unipolar (optical) orthogonal codes with already proposed schemes to design one dimensional optical orthogonal codes and one assumed scheme with ideal results for one dimensional optical orthogonal codes. The proposed schemes of designing one dimensional optical orthogonal codes is very close to ideal one but with higher computational complexity. The second conclusion drawn from the comparison of proposed two dimensional unipolar (optical) orthogonal with already proposed schemes to design two dimensional optical orthogonal codes and one assumed scheme with ideal results for two dimensional optical orthogonal codes. The proposed scheme of designing two dimensional optical orthogonal codes is very close to ideal one but with higher computational complexity. The third conclusion drawn from comparison of proposed two dimensional unipolar (optical) orthogonal codes with proposed one dimensional unipolar (optical) orthogonal. The cardinality of the set of two dimensional optical orthogonal codes is much better than the set of one dimensional optical orthogonal codes of same temporal length and code parameters at the cost of computational complexity. The design of three dimensional and multidimensional optical orthogonal codes may be taken as future work. The challenge is to reduce the computational complexity of the schemes.

The designed one dimensional unipolar (optical) orthogonal codes can be utilized for direct sequence incoherent optical CDMA system to access the optical fiber in asynchronous manner by multiple users. The designed two dimensional unipolar (optical) orthogonal codes can be utilized for wavelength hopping time spreading optical CDMA system with increased cardinality and spectral efficiency. The multiple sets of these codes are designed. It provides flexibility for selection of set of unipolar orthogonal codes with maximum cardinality. The code set with maximum cardinality provides flexibility for selection of unipolar orthogonal codes from same set.

The multiple access interference or probability of error is directly proportional to correlation constraints (λ_a, λ_c). The multiple access interference can be minimized by setting the value of ($\lambda_a=1, \lambda_c=1$) but compromise with lower cardinality or maximum number of codes generated in the set. While with increasing values of correlation constraints ($1 < \lambda_a < w, 1 < \lambda_c < w$), the cardinality of the system can be increased but with the cost of orthogonality which increases the MAI or probability of error or BER.



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Dedicated to

My Human Values Teacher
(Shri Ganesh Prasad Bagaria)

&

My Parents
**(Shri Kailash Narain Singh) &
(Smt. Nirmala Singh)**



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List of Symbols and Abbreviations

Symbol/Abbreviation	Representation/ Explanation
n	Code length
w	Code Weight
λ_a	Auto-correlation Constraint
λ_c	Cross-correlation Constraint
$L \times N$	Matrix dimensions of 2D code
L	Spectral length of Matrix code
N	Temporal length of Matrix code
Z	Size of the code set
J_A	Johnson Bound A
J_B	Johnson Bound B
J_C	Johnson Bound C
1-D	One Dimensional
2-D	Two Dimensional
3-D	Three Dimensional
1-DUOC	One Dimensional Unipolar Orthogonal Code
2-DUOC	Two Dimensional Unipolar Orthogonal Code
AM	Amplitude Modulation
AWG	Arrayed Waveguide Grating
BCDD	Balanced Codes for Differential Detection
BER	Bit Error Rate
BIBD	Balanced Incompleted Block Design
BPSK	Bipolar Phase Shift Keying
BS	Binary Sequence
BW	Band-width



CDM	Code Division Multiplexing
CDMA	Code Division Multiple Access
CHPC	Carrier Hopping Prime Code
DPSK	Differential Phase Shift Keying
DoP	Difference of Positions
DoPR	Difference of Positions Representation
DWDM	Dense Wavelength Division Multiplexing
EDoP	Extended Difference of Positions
EDoPR	Extended Difference of Positions Representation
ECHPC	Extended Carrier Hopping Prime Code
FBG	Fiber Bragg Grating
FM	Frequency Modulation
FTTH	Fiber To The Home
FWPR	Fixed Weighted Position Representation
Gbps	Giga bits per second
GF	Galois Field
LAN	Local Area Network
LASER	Light Amplification by Stimulated Emission of Radiation
MAI	Multiple Access Interference
Mbps	Mega bits per second
MW OOC	Multi Wavelength Optical Orthogonal Codes
$O(x)$	Order of x
OC	Optical Combiner
OCDM	Optical Code Division Multiplexing
OCDMA	Optical Code Division Multiple Access
OOC	Optical Orthogonal Codes
OOK	On-Off Keying
OS	Optical Splitter
OSC	Optical Star Coupler



OTDM	Optical Time Division Multiplexing
OTDMA	Optical Time Division Multiple Access
PG	Projective Geometry
PPM	Pulse Position Modulation
PON	Passive Optical Network
PSK	Phase Shift Keying
QoS	Quality of Service
QPSK	Quadrature Phase Shift Keying
QR	Quadratic Residues
SAC	Spectral Amplitude Coding
SOP	State of Polarization
Tbps	Tera bits per second
TDM	Time Division Multiplexing
TFF	Thin Film Filter
UOC	Unipolar Orthogonal Codes
U(O)OC	Unipolar (Optical) Orthogonal Codes
WDM	Wavelength Division Multiplexing
WHTS	Wavelength Hopping Time Spreading
WP	Weighted Positions
WPR	Weighted Position Representation



CHAPTER 1

1. INTRODUCTION:

In the present age of communication, people from different regions, cultures and languages are coming closer through different mediums of communication to share their knowledge and information. These communication systems are being developed by scientists and technocrats from all across the world. Entrepreneurs, scientists and technocrats are keen to develop systems providing faster communication with higher capacity and bandwidth. They wish to build the cheapest way to provide better quality of services so that every person can afford to communicate with others in the world to share their information and knowledge. This information could be anything ranging from data to audio and video.

Every day better ideas are being implemented to fulfill the basic desire of people to have better communication medium. Now-a-days, the common mediums for communication are Internet, telephone (mobile phone), television and AM/FM radio. These mediums of communication are either wired or wireless i.e. the transmitters and the receivers are connected with each other through cable (wires) or through a wireless medium. The wireless medium may be atmosphere or tropospheric layers which reflects the radio waves with limited bandwidth (Mega-Hertz range) and power. The other mediums providing wireless communication are based on human made satellites which can provide faster communication limited up to few Mbps through stations or towers on earth.

There is another medium which uses optical signals with huge bandwidth, of the order of Tera-Hertz, and it can provide faster communication. Optical transport can be wireless as well as wired. The optical wireless communication is done by highly directional laser beams limited by the line of sight. Optical wired medium i.e. optical fiber can provide communication to a much higher distance with higher capacity and higher speed. The optical cables between two locations have many optical fibers, thus increasing the capacity of optical channel up to thousands of Tera-Hertz. These optical channels can be shared by thousands of users at the same time without any interference. The optical channels with such huge bandwidth started attracting the researchers to explore this communication medium. A lot of hurdles have been resolved by research community since 1970 but new challenges and limits are still being faced [1].

This thesis pertains to one such challenge in optical communication and proposes a better solution. The huge bandwidth of optical fiber can be simultaneously accessed by multiple users using optical multiplexing schemes. In this thesis, the optical code division multiplexing is being investigated as a mechanism to access the optical fiber bandwidth by multiple users. In this scheme, every user is assigned an optical orthogonal code (OOC) or unipolar orthogonal code (UOC) from a code set. Every user's binary information is spread spectrum modulated using the unipolar (optical) orthogonal code assigned to it. Information from all users is multiplexed in the same frequency band after being modulated with the assigned codes. At the receiver side original



information is extracted from the received multiplexed signal by correlating it with the transmitter's signature sequence (unipolar (optical) orthogonal code). This thesis pertains to the algorithms to find multiple sets of one dimensional as well as two dimensional unipolar (optical) orthogonal codes. The algorithms have been compared with existing as well as a hypothetical ideal scheme for designing unipolar (optical) orthogonal codes.

In this chapter, the historical perspective of optical code division multiple access (OCDMA) and optical CDMA codes or unipolar (optical) orthogonal codes as well as motivation and introduction to the research problem are given. The historical perspective is organized into two subsections. First subsection gives the evolution of one dimensional unipolar (optical) orthogonal codes and optical CDMA employing one dimensional codes with fixed length and weight. This subsection also deals with the development of multi-length, multi-weight unipolar (optical) orthogonal codes and Optical CDMA employing these codes. The second subsection deals with the evolution of two dimensional unipolar (optical) orthogonal codes and Optical CDMA employing these codes. This subsection also gives the development of three and multi-dimensional unipolar (optical) orthogonal codes and Optical CDMA employing these codes.

1.1 Historical Perspective:

The history of communication among people is as old as the history of humans. Human beings had communicated with very slow methods transacting small amount of information for thousands of years. The invention of conversion of real time signals into electromagnetic signals as well as their transmission and reception through cables and wireless channels evolved during 1840 to 2013 creating a revolution in human history. It became possible for persons separated by thousands of kilometers, to communicate in real time. This led to immense possibility of collaboration and cooperation between human beings leading to drastic change in social fabric. The electronic wireless channels and man-made satellite for transmission and reception of electromagnetic signals in the range of tens of Mega-Hertz bandwidth provided faster communication with good quality. As time passed, people felt the need of communication which is not only limited to text, audio and video signals among limited number of users but provides high data rate transport to large number of users at same time. One needs a channel with very high bandwidth and capacity to provide high data rate. It is possible upto certain extent through electronic wireless channels. We are already utilizing same electronic wireless channel for communication by multiple users at same time through well known multiple access schemes to fully utilize their capacity. As an alternative, the optical fiber had been a point of attraction for researchers as a medium of communication since 1970 due to availability of huge bandwidth (BW) of the order of tens of Tera-Hertz. The invention of optical fibers (with low attenuation and dispersion) in 1970 by Bell laboratory, LASER sources in 1958 by Charles Townes and Arthur Schawlow, optical amplifiers in 1986 by David Payne and PIN detectors in 1950 by Jun-ichi Nishizawa has revolutionized the research in the area of optical communication. Different techniques to exploit the huge available bandwidth in fiber efficiently have been explored in past. Wavelength division multiplexing (equivalent to frequency division multiplexing in electronics channels) was first introduced in 1970 [2] by O.E. Delange and was realized in laboratory in 1978 by Tomlinson at two wavelengths 1310 and 1550 nm [3]. The



modern system can handle up to 160 signals using DWDM (dense WDM) and thus can expand a basic 10 Gb/s system over a single fiber pair to over 1.6 Tb/s. Another multiplexing known as optical time division multiplexing (optical TDM) was first introduced in 1968 by T. S. Kinsel and R.T. Dinton. In 1988, S. Fujita developed the system employing optical TDM providing data rate of 10 Gbps. Recently, WDM/OTDM transmission systems with a channel rate of 160Gb/s (19 channels), 200Gb/s (7 channels), and 320Gb/s (10 channels) have been reported [4– 6] and the maximum capacity has reached 5Tb/s by using this hybrid scheme [7]. The third and well known multiplexing scheme is code division multiplexing which may be utilized to access an optical channel by multiple users at the same time after spread spectrum modulation of every user's information. The optical CDMA provide all advantages of spread spectrum communication over WDM and OTDMA. The drawback of spread spectrum is overcome here due to huge amount of available bandwidth with optical channel. The story of OCDMA and in turn optical orthogonal codes started around 1980. The evolution of optical orthogonal codes and optical CDMA system can be categorized into following subsections.

1.1.1 Evolution of one dimensional uni-polar (optical) orthogonal codes and optical CDMA systems:

The advantages of CDMA (code division multiple access) system over other multiple access systems are well known to researchers in the field of communication. These advantages forced them to think to access the optical fiber bandwidth using code division multiplexing in optical domain. The Optical CDMA has come across a lot of hurdles and challenges from its inception. The wireless CDMA system requires bipolar orthogonal codes for spread spectrum modulation with binary information of multiple users. But the optical fiber could process only unipolar codes while transmitting the multiplexed information. The design of optical transmitter and optical receiver for CDMA system were big challenges along-with the design of uni-polar orthogonal codes [8]. The researchers accepted the challenges to take advantages of CDMA system to access huge bandwidth of optical fibers.

In 1986, Fan, Prucnal and Santoro gave a basic idea to spread spectrum fiber-optic local area network using optical processing [19]. In 1988, Gagliardi, Khansefid with Taylor proposed a new design of binary sequence sets for pulse coded system [21]. In 1988, Foschini and Vannucci gave the concept of using spread spectrum for making a high capacity fiber optic local area network [22]. In 1989, Salehi. J presented fundamental principles for code division multiple access techniques in optical fiber networks [24, 27]. In 1989, Kiasaleh. K proposed the spread spectrum optical on-off keying communication system [28]. In the same year Gagliardi, Garmire, Kuroda and Mendez proposed a generalized temporal code division multiple access scheme for optical communications [29]. At the end of this year Kwong, Prucnal, and Perrier gave detailed comparison of synchronous versus asynchronous CDMA for fiber-optic LANs using optical signal processing [30, 51]. In 1996, Gagliardi and Mendez gave the performance improvement of optical communications with hybrid WDM and CDMA [72]. In 2002, Sergeant and Stock, described the role of optical CDMA in access network telling merits and demerits of optical CDMA system which makes new challenges in the field of optical CDMA systems [101]. It was a big milestone in this field, with the realities of optical CDMA systems about their physical realization.



The work for design of one dimensional unipolar (optical) orthogonal codes started with the advent of spread spectrum multiplexing. Many researchers had proposed multiple design schemes of unipolar orthogonal codes and their sets. One of these code-sets was proposed by Robinson in 1967 in his research paper [8]. At the same time in 1967 Gold, R. proposed optimal binary sequences for spread spectrum multiplexing [9]. In 1971, Reed proposed a new scheme to generate k^{th} order near – orthogonal codes [10], while in 1979 Shedd and Sarwate proposed another scheme for design of binary orthogonal sequences [11]. The orthogonal binary sequences design was in its early stage and there was a need to convert these binary codes into optical signal. Marom, E in 1978, explained the method to put the optical pulses at the position of bit 1's and no pulses at bit 0's positions of the binary code-word [12]. In 1981, Stark and Sarwate developed the pseudo orthogonal sequences named Kronecker sequences for spread-spectrum communications [14]. In 1983, Davies and Shaar proposed a well known optical orthogonal code design scheme based on Prime sequences and gave an scheme for asynchronous multiplexing for an optical-fiber local area network [15 -16]. This work was a milestone in design of one dimensional optical orthogonal codes using simple mathematics of prime numbers. In 1987, Heritage, Salehi and Weiner proposed frequency domain coding of femto-second pulses for spread spectrum communications [20]. In 1989, Chung, Salehi and Wie proposed another milestone research work on optical orthogonal codes for its basic design, analysis and application [26]. In 1990, Chung and Kumar explained the new bounds for optical orthogonal codes and an optimal construction of these codes [45]. A.E. Brouwer, J.B. Shearer, N.J.A. Sloane, and W.D. Smith had proposed a new table of constant weight codes which can be used as optical orthogonal codes [46]. In 1992, Nguyen, Gyorf, and Massey proposed the constructions of binary constant weight cyclic codes and cyclically permutable codes which can be used as one dimensional optical orthogonal code [54]. In the same year, Holmes and Syms proposed All-optical CDMA using “quasi-prime” codes which was a milestone work related with prime codes [55]. In 1993, Maric, Kostic, and Titlebaum proposed a new family of optical code sequences to be used in spread-spectrum fiber-optic local area networks [56, 57].

In 1994, Kwong, Zhang and Yang proposed 2^n prime sequence codes and its optical CDMA coding architecture [59]. In 1995, Argon and Ahmad [64] proposed optimal optical orthogonal code design using difference sets and projective geometry. Choudhary, Chatterjee, and John had proposed new code sequences for fiber optic CDMA systems [65, 91]. These new code sequences were based on table of prime, quadratic residues and number theory. Bitan and Etzion had proposed constructions of optimal constant weight cyclically permutable codes based on difference families [66]. In 1996, Zhang had proposed strict optical orthogonal codes for purely asynchronous code division multiple access applications [77]. In 2001, Choudhary, Chatterjee, & John proposed one dimensional optical orthogonal codes using hadamard matrices [90]. In 2002, Keshavarzian and Salehi proposed optical orthogonal code acquisition in fiber-optic CDMA systems via simple serial-search method [96]. Lam had proposed symmetric prime-sequence codes for all-optical code division multiple access local area networks [103]. Moschim and Neto proposed some optical orthogonal codes for asynchronous CDMA systems [104]. In 2003, Moreno, Kumar and Omrani provided new construction for optical orthogonal codes, distinct difference sets and synchronous one dimensional optical orthogonal codes [112]. In 2004, Djordjevic and Vasic had proposed combinatorial construction of optical orthogonal codes for OCDMA systems [125] a milestone work. In 2007, Oscar Moreno, Reja Omrani and P. Vijay



Kumar proposed a generalized Bose Chowla family of optical orthogonal codes and distinct difference sets [140]. This construction is optimal with respect to the Johnson bound and can be termed as a milestone work. In 2010, Masanori Sawa proposed optical orthogonal signature pattern codes with maximum collision parameter 2 and weight 4 [150]. In 2010, R.C.S Chauhan, R. Asthana and Y.N. Singh (authors of this thesis) [148,149] had proposed a general algorithm to design sets of all possible one dimensional unipolar (optical) orthogonal codes of same code length and weight. This scheme not only generates one set with maximum number of codes or theoretical upper bound of set but all possible such set with upper bound cardinality for general values of code length ' n ', weight ' w ' and correlation constraints. In 2011, R.C.S. Chauhan and R. Asthana proposed an unique representation named by difference of positions representation (DoPR) and simple calculation of auto-correlation and cross-correlation constraint of one dimensional unipolar orthogonal codes based on DoPR [155].

To overcome the challenges mentioned in [101], some researchers had proposed one dimensional optical orthogonal of multi-length [100] to provide multi-rate optical CDMA system and multi-weight [71, 128, 151, 153] to provide multi QoS in optical CDMA systems. In 1995, G. C. Yang and T. E. Fuja proposed one dimensional optical orthogonal codes with unequal auto- and cross-correlation constraints [70]. In 1996, G. C. Yang, also proposed variable weight optical orthogonal codes for CDMA networks with multiple performance requirements [71]. In 2002, Kwong and Yang proposed designing of multi-length optical orthogonal codes for optical CDMA multimedia networks [100]. In 2005, F. R. Gu and J. Wu proposed construction and performance analysis of variable-weight optical orthogonal codes for asynchronous optical CDMA systems [128]. In 2010, D. Wu, H. Zhao, P. Fan, and S. Shinohara proposed optimal variable-weight optical orthogonal codes via difference packing [151]. In 2011, M Buratti, Y Wei, D. Wu proposed relative difference families with variable block sizes and their related OOCs to design variable weight optical orthogonal codes [153]. In 2012, R.C.S Chauhan, Y.N. Singh and R. Asthana had proposed another search method using clique search algorithms to find multiple sets of one dimensional unipolar orthogonal codes for given code parameters in very efficient manner [156].

As soon as basic fundamentals of optical cdma are becoming more clear along with some schemes which were proposing design of one dimensional orthogonal codes, some researchers started studying the performance analysis of optical cdma system employing these codes. In 1981, Weber and Batson gave the performance analysis of code division multiple access system employing pseudo orthogonal sequences [13]. In 1985, Tamura and Okazaki gave the analysis related to optical code division multiplexed transmission by employing the Gold sequences [17]. In 1988, MacDonald, R.I. proposed fully orthogonal optical code division multiplexing for broadcasting [23]. In 1989, Brackett with Salehi gave system performance analysis of the code division multiple access techniques in optical fiber networks [25]. In 1990, Gagliardi, Khansefid and Taylor had proposed performance analysis of code division multiple access techniques in fiber optics with on-off and PPM pulsed signaling [47]. In 1994, Walker gave a theoretical analysis of the performance of code division multiple access communications over multimode optical fiber channel. There are two parts of his analysis, part-I showing transmission and detection and part-II with system performance evaluation [60, 61]. In 1995, Yang, G. C. and Kwong, W. C. employed prime codes [15] to study the performance analysis of



optical CDMA system [69]. Kavehrad and Zaccarin proposed optical code-division-multiplexed systems based on spectral encoding of non-coherent sources [63]. In 1996, Ho, C.L. described the performance analysis of optical CDMA communication systems with quadratic congruential one dimensional optical orthogonal codes [76]. In 2002, Argon and McLaughlin gave the comparative study of optical OOK-CDMA and PPM-CDMA systems with turbo product codes [102]. Forouzan, Kenari and Salehi had proposed frame time-hopping fiber-optic code-division multiple-access using generalized optical orthogonal codes [105]. In 2007, P. Saghari, R. omrani, with P.V. Kumar had proposed a scheme of increasing the number of users in an optical CDMA system by pulse position modulation [142].

Some of the researchers are doing experimental demonstration of the optical cdma systems. In 1991, Macdonald and Vethanayagam demonstrated a novel optical code division multiple access system at 800 mega-chips per second [50]. In 1994, Gagliardi and Mendez gave synthesis of high speed and bandwidth efficient optical code division multiple access and its demonstration at 1Gb/s throughput [62]. In 2002, Sotobayashi, Chujo and Kityama had demonstrated 1.6-b/s/Hz, 6.4-Tb/s QPSK-OCDM/WDM (4 OCDM X 40 WDM X 40 Gb/s) transmission using optical hard thresholding [97].

1.1.2 Evolution of Two and multi-dimensional uni-polar (optical) orthogonal codes and optical CDMA systems:

Researchers started to explore the optical orthogonal codes and optical CDMA system in more than one dimension [48, 53, 58, 67] to take advantages of multidimensional over one dimensional OOC [133] like good spectral efficiency without cost of affecting BER performance. In 1992, Garmire, Mendez and Park gave design and demonstration for temporal/spatial optical CDMA network and comparison with temporal networks [53]. In 1996, Tancevski, and Andonovic, described hybrid wavelength hopping/time spreading schemes for use in massive optical networks with increased security [74, 75]. In 1998, Deppisch and Elbers proposed coarse WDM/CDM/TDM concept for optical packet transmission in metropolitan and access networks supporting 400 channels at 2.5 Gb/s peak rate [87]. In 2001 Yegnanarayanan, Bhushan and Jalali proposed fast wavelength hopping time spreading encoding / decoding for optical CDMA [88]. In 2006, Reja Omrani and P. Vijay Kumar proposed an overview of one dimensional and two dimensional optical orthogonal codes as well as some new results relating to bounds on code size and code construction [133]. Ken-ichi Kityama, Xu Wang, and Naoya Wada described optical CDMA over WDM PON – Solution path to Gigabit-Symmetric FTTH [136].

Some researchers started exploring the optimal design of two dimensional optical orthogonal codes. In 1993, Gagliardi and Mendez developed the matrix or two dimensional optical orthogonal codes for ultra-dense Giga-bit optical CDMA networks [58] a milestone work. In 1996, Yang and Wong proposed two-dimensional spatial signature patterns [73]. In 1997 Iversen, Jugl and Kuhwald proposed an algorithm for construction of unipolar (0,1) matrix codes [78]. In 1998, Selvarajan, Shivaleela and Shivrajan proposed a new design of new family of two dimensional codes for fiber optic CDMA networks [80] a milestone work. In 2002, Lee and Seo proposed new construction of multi-wavelength optical orthogonal codes [106]. In 2003,



R.M.H.Yim, Jan Bajcsy, L.R.Chen had described a new family of 2-d wavelength-time codes for optical CDMA with differential detection [113]. Pu, Li and Yang researched algebraic congruent codes used in two dimensional optical CDMA system [114]. In 2004, Yang and Kwong provided a new class of carrier-hopping codes for code division multiple-access optical and wireless systems [115, 126] a milestone work. Griner and Arnon had proposed a novel bipolar wavelength-time coding scheme for optical CDMA systems [119]. Yang, Kwong and Chang had provided multiple-wavelength optical orthogonal codes under prime-sequence permutations [120]. Kwong and Yang also proposed extended carrier-hopping prime codes for wavelength-time optical code-division multiple accesses [127]. Both [126, 127] of them are milestone works in design of two dimensional OOC.

Some of the researchers felt need of unique representation of two dimensional optical orthogonal codes and simple calculations of auto-correlation and cross-correlation constraints in parallel with exploring designing methods of two dimensional unipolar orthogonal codes. In 2006, Hossein Charmchi with Jawed A. Salehi had proposed outer-product matrix representation of two dimensional optical orthogonal codes instead of applying commonly used approaches based on inner product to construct optical orthogonal codes [138]. In 2009, H. Cao and R Wei proposed combinatorial construction for optimal two dimensional optical orthogonal codes [146]. In 2011, Y C Lin, G C Yang, and W C Kwong given construction of optimal 2D optical codes using $(n, w, 2, 2)$ optical orthogonal codes [154]. In 2013, R C S Chauhan, Y N Singh and R Asthana proposed not only a scheme to design two dimensional unipolar (optical) orthogonal codes through one dimensional unipolar (optical) orthogonal codes but also an unique representation and simple method to calculate correlation constraints of two dimensional unipolar orthogonal codes [157].

After having basic knowledge of wavelength hopping time spreading optical cdma system and two dimensional or matrix orthogonal codes, some researchers started studying performance analysis (BER and spectral efficiency) of WHTS optical cdma system employing 2D OOC. In 1991, Kiasaleh, K. proposed fiber optic frequency hopping multiple access communication system employing two dimensional optical orthogonal codes [48]. In 1995, Andonovic and Tancevski developed hybrid wavelength hopping time spreading code division multiple access systems employing two dimensional optical orthogonal codes [67]. In 2001, Hu and Wan proposed two dimensional optical CDMA differential system with prime optical orthogonal codes [93]. In 2002, Yim Chen and Bajcsy proposed design and performance of 2-D codes for wavelength-time optical CDMA [98]. In 2003, Mendez, Gagliardi, Hernandez, Bennett and Lennon provided the design and performance analysis of wavelength/time (W/T) matrix codes for optical CDMA [111]. In 2004, Mendez, Gagliardi, Hernandez, Bennett and Lennon proposed high performance optical CDMA system based on 2-D optical orthogonal codes [121].

In this period of 2005-2010, researchers of this stream was mainly focusing on interference avoidance, increasing spectral efficiency and improving the security performance of optical CDMA system. In 2005, Yang and Kwong given the performance analysis of extended carrier-hopping prime codes for optical CDMA [129]. Kwong, Yang and Chang proposed wavelength hopping time spreading optical CDMA with bipolar codes [130]. Kutsuzawa and Minto proposed a field demonstration of time spread/wavelength –hop OCDM using fiber Bragg



grating encoder/decoder [131]. E. S. Shivaleela, A. Shelvarajan, and T. Srinivas proposed two dimensional optical orthogonal codes for fiber-optic CDMA networks [132]. In 2006, Sun Shurong, Hongxi Yin, Ziyu Wang and Anshi Xu proposed a new family of 2-D optical orthogonal codes and analysis of its performance in optical CDMA access networks [137]. In 2007, Xu Wang, Naoya Wada with Ken-ichi Kityama [141] proposed 111 km error free field transmission of asynchronous 3-WDM x 10-OCDMA x 10.71-Gbps with differential phase shift keying for data modulation and balanced detection [141] which may be termed as big milestone work in realization of two dimensional optical CDMA system.

The historical perspective over the development of 3D optical orthogonal codes is as follows. In 1991, Gagliardi and Mendez proposed performance analysis of pseudo orthogonal codes in temporal, spatial and spectral code division multiple access systems [48]. In 2000, Kim, Kyungsik and Park proposed a new family of space / wavelength / time spread three dimensional optical orthogonal codes for optical CDMA networks [84]. McGeehan, Nezam, Omrani and Kumar proposed three dimensional time-wavelength-polarization OCDMA coding for increasing the number of users in OCDMA LAN [118]. In 2010, J Singh and M L Singh proposed design of 3-D wavelength/time/space codes for asynchronous fiber optic CDMA system [152] which may be termed as beginning in the field of three dimensional OOC. N. Tarhuni, M. Elmusrati and T. Korhonen proposed polarized optical orthogonal code for optical CDMA systems, by exploiting the polarization property of the fiber and the chip's polarization state which may be treated as third dimension of the code [134] is a milestone work.

1.2 Optical CDMA Systems:

1.2.1 Introduction

The Optical code division multiple access (CDMA) is a scheme of accessing the optical channel by multiple users simultaneously. Every user of optical CDMA system has been assigned one individual signature sequence or optical orthogonal code from the same set of optical orthogonal codes. The user spreads binary data by spread spectrum modulation with the assigned optical orthogonal code. All users' spread spectrum information gets code division multiplexed before transmission over the optical channel. The multiplexed information from the channel is received and gets correlated with the authorized signature sequence or optical orthogonal code at dedicated receiver. The original information is extracted only at those receivers which have the same optical orthogonal codes as assigned to multiple users accessing the channel. The cardinality or maximum number of users of OCDMA system, is always less than or equal to the cardinality of the set of orthogonal codes used for assignment [1, 23, 29]. The Optical CDMA accesses the optical channel either in asynchronous or synchronous way. The asynchronous access of channel by multiple users makes the system free from the centralized control so that any user can start accessing the channel at any time with its assigned code sequence. While the synchronous access of channel requires centralized control so that every user could send information at some specific time. In asynchronous or synchronous optical CDMA system, all users accessing the channel are connected through optical star coupler (OSC). In a network span, there may be two or more than two such optical star couplers connected with each other via optical channels and couplers as in figure 1.1. Each user is connected to OSC and



has a transmitting as well as receiving unit with different optical orthogonal codes as shown in figure 1.2.

The code assigned to receiver unit is the same code as assigned to the transmitter from which the receiver unit is to receive the information. Every transmitting unit send optical signal to nearby star coupler to get code division multiplexed with optical signals from other users connected to the same star coupler. Similarly, every receiving unit receives the code division multiplexed optical signal from nearby star coupler to de-multiplex and decode the information at the destination. The way of multiplexing of optical signals at star coupler decides about asynchronous or synchronous access of channel. In synchronous optical CDMA, the optical signal from every transmitter unit get code division multiplexed with synchronized data bits from every user. While in asynchronous optical CDMA, the optical signal from every transmitter unit get code division multiplexed with synchronized optical pulses or optical chips not necessarily data bits from every user [30].

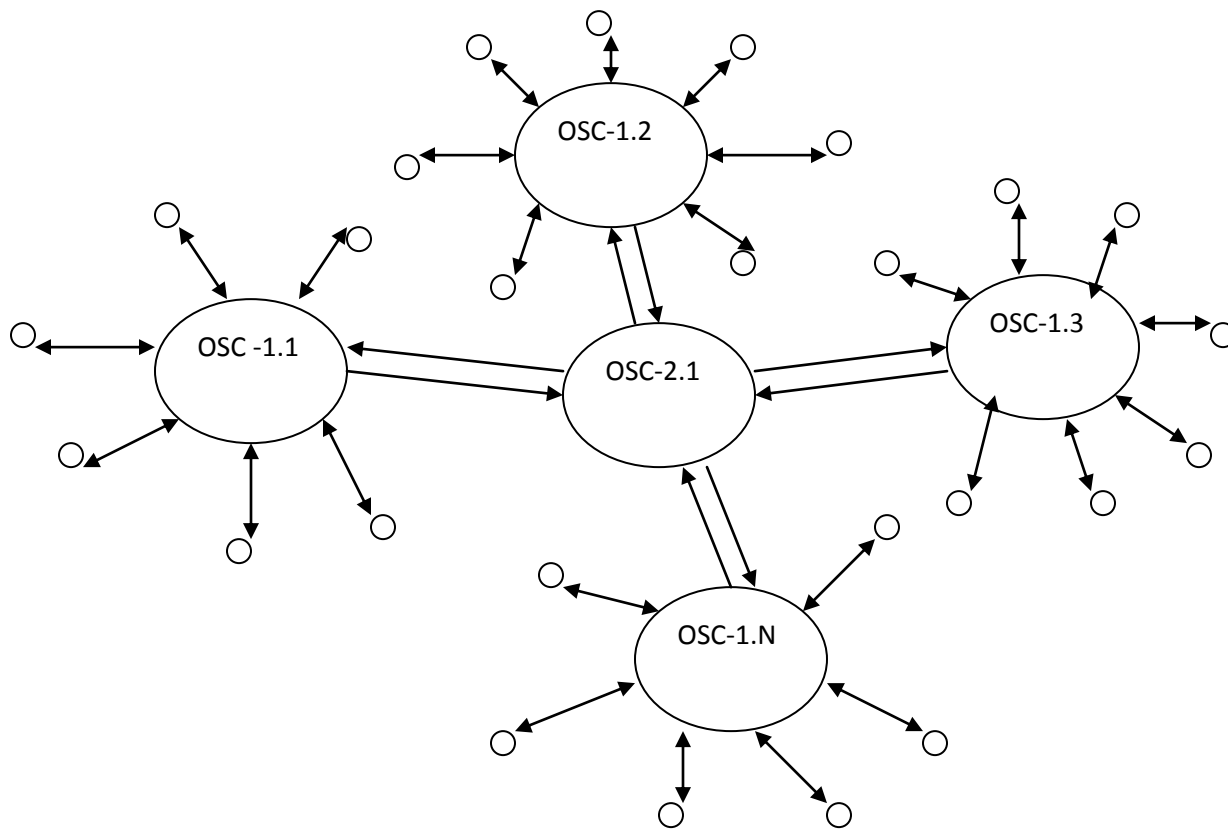


Figure 1.1: Optical CDMA network



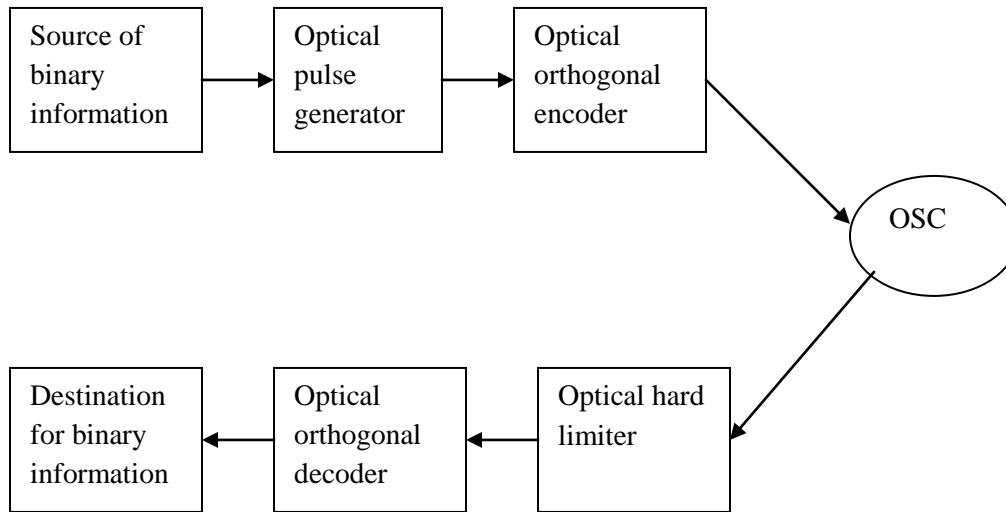


Figure 1.2: optical CDMA user with transmitter and receiver section

Every transmitting unit is equipped with source of binary information, optical pulse generator and optical orthogonal encoder or optical spread spectrum modulator. Every receiving unit is equipped with optical hard limiter, optical orthogonal decoder and destination for received binary information. Conventionally in transmitter section optical pulse generator generates a coherent optical pulse of narrow width for data bit '1' and no pulse for data bit '0' following on-off keying modulation. This on-off keying modulation may be replaced by pulse position modulation (PPM) or phase shift keying (PSK) modulation. The optical orthogonal encoder consists of optical splitter (OS), filter with optical delay lines and optical combiner (OC). The filter with optical delay lines is designed as per the optical orthogonal code assigned to particular transmitter section as shown in figure 1.3a. Let the optical orthogonal code with code length ' n ', and code weight ' w ' or ' w ' positions of bit '1' spread over code length ' n '. The optical delay line filter consists of ' w ' parallel optical delay lines [12, 18] with delays equal to position of bit '1's of the code assigned to the transmitter section. For example the code with code length $n=7$, code weight $w=3$, with weighted positions (1, 2, 4) so that optical delay lines with delays equal to (1, 2, 4) respectively. At the receiver section the optical hard limiter receives the multiplexed pulse information and limits them at fix level of amplitude.

The optical orthogonal decoder consists of optical splitter, matched delay line filter, optical combiner and optical threshold detector. The optical splitter receives the multiplexed optical pulses and sends them to every line of matched delay line filter. The matched delay line filter consists of $w=3$ parallel optical delay lines with delays decided by the position of bit '1's of the particular code assigned to the receiver section as ($n-1=6$, $n-2=5$, $n-4=3$) shown in figure 1.3b. The optical combiner receives the optical pulses from every delay line of filter and arranged them in order. The pulses received at same time are overlapped with each other to be detected by threshold detector for auto-correlation peak. The auto-correlation peak within bit duration is detected as bit '1' by threshold detector while no auto-correlation peak within bit duration is detected as bit '0'.



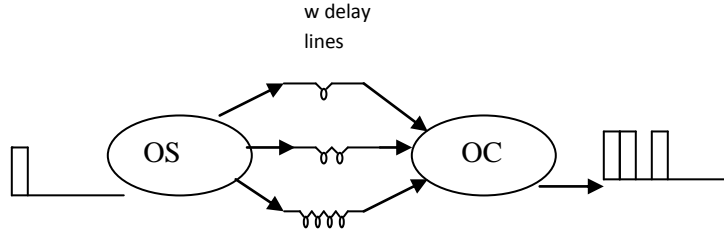


Figure 1.3a. Optical orthogonal encoder for code length $n = 7$, weight $w = 3$ with weighted positions at (1,2,4). (OS – Optical Splitter), (OC – Optical Combiner).

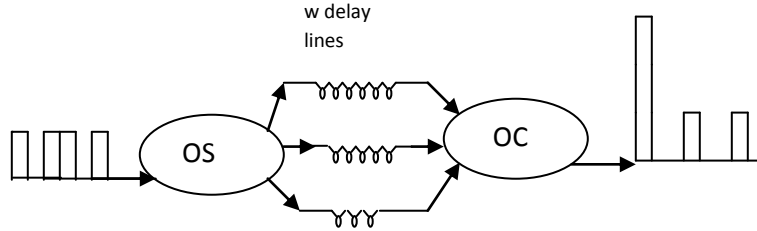


Figure 1.3b. Optical orthogonal decoder for the code length $n = 7$, with weight $w = 3$ at weighted positions (1,2,4) and delays ($n-1=6$, $n-2=5$, $n-4=3$).

1.2.2 Types of Optical CDMA Systems based on Optical Coding Techniques:

The optical delay line filter is used to generate incoherent optical pulse signal for a particular optical orthogonal code and data bits. This optical delay line filter can be replaced by another filter generating M-ary coherent optical pulse signal for particular optical orthogonal codes and data bits. Suppose for $M=2$, bit '1's of the code are replaced by optical pulses with 0 phase difference while all '0's within code are replaced by optical pulses with phase difference ' π ' to generate coherent bipolar phase shift keying (BPSK) optical signal. On the basis of generating incoherent and coherent optical pulse signal for data bits and particular optical orthogonal codes, the optical CDMA can be categorized into incoherent optical CDMA and coherent optical CDMA. For incoherent optical CDMA, only unipolar (optical) orthogonal codes can be employed as optical signature sequence for spread spectrum modulation. While for coherent optical CDMA system unipolar as well as bipolar optical orthogonal codes can be employed as optical signature sequence for spread spectrum modulation. But the cardinality of set of bipolar orthogonal codes is higher than the set of unipolar orthogonal codes for code length ' n '. The cross-correlation of bipolar orthogonal codes is always zero, therefore, generally bipolar orthogonal codes are used in coherent optical CDMA system.

1.2.2.1 Incoherent Optical CDMA Systems:

As we know that the spread spectrum modulation of an optical pulse with unipolar (optical) orthogonal code generates the optical signal with incoherent pulses. In an Incoherent Optical CDMA system, each user is assigned a unique unipolar (optical) orthogonal code which is used by other user as signature sequence to send the information to the authorized user of that signature sequence. The information bit '1' is sent by sending the signature sequence of



authorized user for the bit period T_b , while bit '0' is represented by no optical pulses for the bit period T_b . The problem of collision arises if two or more than two users are trying to send their information to the same destination with unique address or signature sequence. This can be solved by setting a central distributor of signature sequences to the active user from a specific code set and keeping the information that which code is busy and which is free. Any active user first checks for availability of destination by looking the status of codes available and then process the information to avoid the problem of collision mentioned above. The incoherent encoding for Optical CDMA can be done in following ways.

(a) Temporal Spreading:

It is the very first and well known incoherent coding schemes, in which the data bit period T_b is divided into ' n ' chip time intervals. Here ' n ' is the length of one dimensional unipolar (optical) orthogonal code. The optical pulsed signal is created by putting the optical pulses at the weighted chips by bit '1's in the code, so that optical pulses present in a bit period T_b are equivalent to weight of the code. The once generated optical pulsed signal is sent to the optical star coupler (OSC) for information bit '1', while the code of zero weight and same code length n is sent for bit period T_b for information bit '0' [23,24] for code division multiplexing at OSC. All other active users are also sending their optical pulsed signals or (temporally spread signal) generated with their assigned optical orthogonal codes in every bit period T_b to OSC for multiplexing with others. At every authorized receiver the original information is decoded with assigned optical orthogonal code. It should be very clear that all users use zero weight code for sending information as bit '0'. If the cross correlation of received signal and assigned code is less than the cross correlation constraint, the decision is taken as bit '0' for bit period T_b and if cross-correlation is greater than correlation constraint, the decision is taken in favor of bit '1' in time duration T_b . The received information with all '0' bits has no meaning at any destination.

A lot of schemes are proposed for designing of sets of one dimensional unipolar (optical) orthogonal codes in literature. A suitable set of codes can be selected for assigning of optical orthogonal codes to distinct users of temporally spread incoherent optical CDMA systems. The limitations of temporal spreading are requirement of long codes and short optical pulses for good correlation properties [121].

(b) Spectral Amplitude Coding (SAC):

In the SAC-OCDMA system, the source spectrum is assumed to be flat over a bandwidth and the transmitted spectrum is divided into ' n ' rectangular slices which are amplitude masked as per the optical orthogonal sequence of the user by the use of diffraction grating and spatial masks [79]. The coded spectrum and its complement are propagated for transmitting the binary information '1' and '0' respectively [63]. The unipolar SAC codes can be generated with proposed schemes in the literature for design of one dimensional unipolar (optical) orthogonal codes to improve the performance of the SAC-OCDMA system. The optical encoded information as per their codes from other users is passed to star coupler for multiplexing. This code division multiplexed information reaches to all the active receivers through single mode optical fibers. At each receiver the information is decoded using wavelength splitter and wavelength combiners for the known optical orthogonal codes [79].



(c) Spatial Coding:

The Spatial coding can be employed with temporal spreading and / or spectral amplitude coding to advance the coding into two or three dimensional optical coding in multiple fiber system using fiber tapped delay lines for encoding and decoding [1]. The multimode fibers can also be employed for spatial techniques using 2D spatial masks for encoding the specific speckle patterns as code sequences [73, 1]. The use of spatial coding is limited by the requirement of multiple star couplers and equal optical path length from encoder/decoder to the couplers.

(d) Wavelength Hopping Time Spreading:

The Wavelength Hopping Time Spreading (WHTS) OCDMA system uses the 2D optical orthogonal codes to spread the coded information in time and wavelength domains simultaneously. The two dimensional optical orthogonal codes or WHTS codes can be generated in matrix form by the proposed schemes in literature [53, 58, 73, 80, 88, 93, 98, 106, 111, 113, 114, 119, 120, 121, 126, 127, 132, 137] also known as matrix codes. The WHTS codes can be implemented with multi-wavelength Laser source hopping from one wavelength to another very rapidly. The encoder uses w specific wavelengths pulses, to position them at weighted chips within bit period T_b . These WHTS encoders are based on arrayed waveguide gratings (AWG) and thin film filters (TFF) while linear array of fiber Bragg gratings (FBG) based encoder require complex schemes for independently delaying each wavelength [1]. The WHTS decoders are also implemented with AWGs and TFFs [1]. The function of WHTS decoder is to discriminate between desired and interfering signals by using the correlation of received signal with assigned WHTS signature sequence or matrix code. If the received signal is matched with assigned matrix code at each wavelength, the amplitude of autocorrelation peak becomes equal to weight ' w ' of the matrix code. While the unmatched matrix code generate the Multiple access Interference (MAI) at the correlator output.

The 2D WHTS codes have better spectral efficiency as compared to the temporally spread one dimensional optical orthogonal codes over wavelength division multiplexing (WDM) in incoherent optical CDMA systems [122].

1.2.2.2 Coherent Optical CDMA Systems:

In the coherent optical CDMA system, the phase shift keying or phase coding of the optical pulsed signal is used with the assigned orthogonal code to the user. The optical pulsed signal is derived from highly coherent wideband source such as mode locked laser. The coherent Optical CDMA receiver section is made synchronous with transmitter section so as coherent reconstruction of user's data is possible. This coherent transmission and reconstruction is also possible with Polarization shift keying of the optical signal field over user's code. On the basis of coherent encoding schemes, the Coherent Optical CDMA system can be classified into following schemes.

- a. Temporal Phase Coded Optical CDMA
- b. Spectral Phase Coded Optical CDMA
- c. Polarization encoded Optical CDMA



(a) Temporal Phase Coded Optical CDMA

In Temporal Phase Coded Optical CDMA system [91], the mode-locked laser with short pulse capabilities is used to generate mode locked pulses to modulate the user data stream with On-Off keying format, DPSK or Duo-binary or any other complex modulation format at each encoder [1]. The temporal phase encoder creates ' n ' pulse copies of modulated pulse output with T_c chip interval between any two consecutive pulses, in a bit period T_b . These ' n ' pulse copies are set with a specific relative phase shift depending on the user's code. The specific relative phase shift is determined by simple binary code (1,0) such as 0, π or M –ary phase shift keying [82]. To decode the information, the receiver is employed with time domain matched filter to perform temporal correlation of the ' n ' copies of the received signal with appropriate temporal phase code. The received signal is delayed by ' n ' delay elements, each of jT_c delay for $j = 0$ to $n-1$ so that received signal in each chip interval is multiplied with pulse in corresponding chip interval of the temporal code and then integrated over a bit period T_b [1]. The auto-correlation peak output is obtained, if temporal phase code is matched otherwise cross correlation output will be low, noise like, which is also known as multiple access interference or the effect of presence of other signals.

(b) Spectral Phase Coded Optical CDMA

In Spectral Phase Coded Optical CDMA [122,139], a mode locked laser generating broadband multi-wavelength and highly coherent light in frequency domain is employed. The user data is modulated with continuous pulse output of mode locked laser by on – off keying or any other modulation format as in temporal phase coded optical CDMA. All users are assigned their signature sequences or orthogonal codes from a set of n element spectral phase codes. This modulated data output can be expressed in frequency domain and ' n ' copies of it, are created in a bit period T_b by using diffraction grating or virtual imaged phase array grating or micro ring resonator [1], with the phase difference (0 or π) applied to each spectral elements as per binary (1,0) code of length n bits. In the time domain, it is equivalent to temporal broadening of mode locked laser temporal pulse output, making the encoded signal more like a noise. The optical channel accepts n such encoded output, which is passed, in combined form, to each receiver for decoding purpose. The decoder is employed with the same device as encoder but with conjugate spectral phase coded mask in order to recover the original signal after correlation by matched filtering (time gating) and then optical thresholding [1,122].

(c) Polarization Encoded Optical CDMA:

In Polarization encoded Optical CDMA system, the mode locked laser pulse output is modulated with binary data in on –off or any other modulation format. The modulated data output in a bit period T_b is used to make its ' n ' copies at the interval of chip duration T_c to fully cover the bit duration T_b . Each pulse in the chip period T_c is given either of two state of polarization (SOP) by Polarization Shift Keying as per the binary (0,1) optical orthogonal code assigned to the user. All such n outputs are intermixed into single mode optical fiber to transfer it to all n receivers. At each receiver, the assigned polarized optical orthogonal code is generated and used for correlation to recover original signal after optical thresholding [134].



1.2.3 Multiple Access Interference and Reduction Schemes :

MAI is a common drawback of any CDMA system. it is the interference to the signal due to presence of un-orthogonal signals from other transmitters at each receiver of OCDMA system. This interference is caused in the optical channel due to simultaneous access of same channel by two or more than two optical transmitters with imperfect optical orthogonal codes as signature sequences. This MAI is caused only when at least one other transmitter is sending the imperfect optical orthogonal code for data bit '1', while for data bit '0' no MAI, because for data bit '0' no optical pulses are sent to the channel. Hence the main reason for MAI is use of imperfect orthogonal code as signature sequences to access the optical channel.

For the case of Incoherent Optical CDMA system the optical orthogonal codes are uni-polar. The uni-polar codes are formed from binary digits (0 and 1) with code length ' n ' representing the number of bits in code-word and with code weight ' w ' representing the number of total '1' bits in the code word. As by the definition of perfect orthogonal code the dot product of two or more than two perfect orthogonal code is zero. As per definition of optical orthogonal codes and its auto-correlation properties [25] the uni-polar orthogonal code and its all shifted sequences represent to same uni-polar orthogonal code word. The dot product of these shifted sequences with optical orthogonal code word should be minimized upto zero for proper detection of autocorrelation peak as synchronizing pulse at receiver and detection of data bit '1'. The dot product of orthogonal code word with its shifted sequences can be constraint upto a limit called auto-correlation constraint λ_a which is not responsible for MAI but its higher value close to auto-correlation peak (amplitude w) may be responsible for wrong detection at receiver, hence it should be minimized upto zero. For the case of uni-polar orthogonal code word auto-correlation constraint λ_a can not be less than one because of binary dot product. The auto-correlation constraint λ_a has a range from 1 to $w-1$.

As per definition of optical orthogonal code and its cross-correlation properties [25], the maximum value of dot product of one uni-polar orthogonal code word with other orthogonal code word and its shifted sequences is called the cross correlation constraint λ_c . No uni-polar orthogonal code word pair has λ_c equals to zero because of binary dot product. This cross-correlation constraint λ_c is ranged from 1 to $w-1$. The non zero value of λ_c is responsible for MAI. Hence MAI is always present at the receiver in case of Incoherent optical CDMA system with uni-polar orthogonal codes. The MAI can be minimized by the use of uni-polar orthogonal codes of minimum value of λ_c i.e. one. Similarly for correct detection and synchronization, minimum value of λ_a should be one. Hence we search for the code set of maximum possible optical orthogonal codes with minimum λ_a and λ_c . The maximum possible optical orthogonal code number N of code length ' n ', code weight ' w ' with $\lambda = \lambda_a = \lambda_c$ is given by Johnson's' bounds [25].

The Optical CDMA transmitter sends the optical orthogonal encoded information for data bit '1' and no optical pulses for data bit '0' to the channel. The channel accepts such N signals to



transmit them at every receiver. The receiver correlates this intermixed signal from channel with its signature sequence (assigned optical orthogonal code for data bit detection). If one user is sending data bit '0', i.e. no optical pulses in the bit period T_b , the receiver accepts $N-1$ encoded signal, which is correlated with already stored user's code, if it has a finite value exceeding particular threshold value, it may be detected as bit '1' which is the wrong decision by detector as bit '0' was forwarded. While there is no error if bit '1' was sent, as it is always decoded as bit '1' even in the presence of multiple access interference [23-24]. It can be estimated how the probability of error is related with the MAI and other parameters of the orthogonal code set. The probability of error for chip synchronous and noise free OCDMA [24] is calculated as

$$P(E) = \sum_{i=\mu}^{N-1} \binom{N-1}{i} \left(\frac{w^2}{2L} \right)^i \left(1 - \frac{w^2}{2L} \right)^{N-1-i}$$

Here N is number of active user, μ is the threshold value, w is the weight of the code, and L is T_b/T_c or length of the code word.

To reduce the MAI or $P(E)$ which is responsible for bit error, a lot of schemes are proposed in [34-44] by Researchers time to time for different optical cdma systems. The schemes which ultimately improve the performance of system are being summarized as follows

(i) There is a scheme with modified PN sequences and Fiber Bragg Grating (FBG) encoder and decoder [34], Where the modified PN codes has even number of '1' and '0' in the code of even length. Suppose a modified PN sequence (11100100) is assigned to one user. This user generates a optical stream of different pulses as $(\lambda_1 \lambda_2 \lambda_3 00 \lambda_6 00)$ by FBG encoder for data bit '1', and it's complement as $(000 \lambda_4 \lambda_5 0 \lambda_7 \lambda_8)$ is generated by same FBG encoder for data bit '0'. The receiver is equipped with FBG decoder and two photo diode PD1 and PD2. The received signal is matched by FBG decoder and passes to either of photo diode. The PD1 detects the signal as data bit '1' and PD2 detects the signal as data bit '0'. This proposed system can obtain the same performance at a lower Signal to Interference Noise Ratio (SINR) by 6 dB than that of convention system with uni-polar capacity.

(ii) There is high performance optical thresholding technique is demonstrated by using super-continuum (SC) generation in normal dispersion-flattened-fiber (DFF) for reducing the MAI in high chip rate coherent OCDMA system [35]. The proposed SC is comprised of an EDFA, a 2 km long DFF, and a 5nm band pass filter (BPF). The operating principle is that the decoded optical signal is boosted by EDFA to high peak power with 2 ps pulse width which can generate SC in the DFF. The incorrectly decoded signal is spread over large time span with low peak power so that it is unable to generate SC. The BPF only allow the SC signal and rejects the otherwise so that MAI noise is suppressed.

(iii) The Adaptive Resonance Code (ARC) are generated with a given algorithm in [36] and compared with other Time Wavelength Hybrid (TWH) codes for Multiple Access Interference (MAI). The MAI is most responsible factor for Bit Error Rate (BER).



$$(MAI)_i = \sum_{j=1, j \neq i}^{N-1} \max (R_{xy}(m))$$

$(MAI)_i$ is multiple access interference at i_{th} node (for i_{th} user)

And $R_{xy}(m)$ is cross correlation of two sequences X and Y

As less as MAI at i_{th} node, as less as the bit error rate for that node. The ARC code are selected on the basis of the least value of MAI between two codes as compared with the MAI of selected codes. So that the ARC always present with lowest value of MAI as compared with other code set like OW (OOC + WDMA), PH (Prime Hop), EP (Eqc. Prime), MW (Multi Wavelength) , OC (Optimal Code). The MAI of these different coding schemes is compared in [36] and found ARC with lowest MAI, i.e. with lest probability of error or BER.

(iv) The Interference Avoidance for Optical CDMA system is described in two parts (i) State estimation and (ii) Transmission scheduling [37].

In state estimation the node estimate the state of line and next the transmission scheduling decides the appropriate time to send the packet so that lower number or no collision of weighted chips occur. The packets sent between state estimation and transmission scheduling may causes interference. By the protocol Interference sensing/ Interference detection (IS/ID), the node estimate the state and schedule the transmission. In [38-41], the estate estimation and transmission scheduling are described along with there algorithms.

(v) The serial interference cancellation with first stage in which the interference is reproduced and then subtracted from received data is described in [42]. For N number of users there are N-1 serial cancellation stages along with N-1 different optical orthogonal codes. Here first cancellation stage is described for desired user #1 with consideration of interference produced by N^{th} user with different threshold values starting from 1 to w, the weight of the code.

The BER performance of the described system is compared with conventional optical cdma system for different number of users yielding better performance as compared with others.

(vi) The parallel interference cancellation stage is described with lowest threshold value [43]. Initially BER performance of conventional OCDMA system is compared with OCDMA system with optical hard limiter before the OCDMA receiver. It reduces the effect of MAI upto some level but not the effect of other noises in the receiver at lowest threshold value. This problem can be solved by using parallel interference cancellation stages. All N-1 interference cancellation stages are connected in parallel to get the sum of all interference produced by each respective unwanted users. This complete interference is subtracted from the received signal to get the desired signal without MAI.

(vii) A simple direct detection optical PPM – CDMA system is described with interference cancellation [44]. The modified prime sequences are used as signature sequence, and at receiver the Poisson effects of photo-detection process are considered.



1.3 Motivation & Objectives:

The dream of communication Engineers and research persons is to fully utilize the available bandwidth of optical fibers for errorless communication. To make this dream come true, there are a lot of challenges to be resolved. These may be related with ultra-short optical pulse design, optical pulse modulation schemes, optical code design, optical fiber with lower attenuation over wide band of wavelength of optical signal, optical fiber with zero dispersion, integration of many multimode fibers within an optical cable with zero interference, multiple access interference reduction schemes, optical amplifiers or repeaters, optical detector with lower receiver noises, optical delay line filters, optical decoder or demodulator, optical pulse memories, etc. In this thesis, the authors are motivated to accept and resolve the challenges related to code division multiple access scheme and optical orthogonal code design. The well known schemes for design of one dimensional as well as two dimensional unipolar (optical) orthogonal codes and their multiple sets have been studied. It is found that the schemes proposed in literature [8-11, 14-15, 21, 25, 55-57, 59, 64, 65, 68, 77-78, 90-91, 95-96, 103-104, 112, 125, 133, 138, 140, 145-146] for design of one dimensional unipolar (optical) orthogonal codes are generating one set of these codes. The maximum code set size is given by Johnson bounds [25, 45, 133], but only some of these proposed schemes are generating codes with maximum code size. These schemes are specific for selection of code length ' n ', code weight ' w ', maximum auto-correlation and cross-correlation constraint of the codes within a set. It motivates to explore an scheme which can generate all one dimensional unipolar (optical) orthogonal codes within a set for all general values of code length ' n ' code weight ' w ' and given maximum auto-correlation and cross-correlation constraint of the codes within a set.

It also motivates to explore a general scheme to design all possible sets of maximum number of one dimensional unipolar orthogonal codes for any given value of code length ' n ' code weight ' w ', maximum auto-correlation and cross-correlation constraint of these codes within a set.

In a similar way all well known two dimensional unipolar (optical) orthogonal codes and their multiple sets design schemes [53, 58, 73, 80, 88, 93, 98, 106, 111, 113, 114, 119, 120, 121, 126, 127, 132, 137] are studies. The situation is almost same as in one dimensional orthogonal code design schemes. No scheme is generating a set with maximum number of two dimensional codes. While some of these scheme [84], [118] are trying to design multiple sets of these codes of same matrix size, $L \times N$, weight ' w ' and given maximum auto-correlation and cross-correlation constraint of codes within a set, the designed sets have number of codes less than maximum code set size. The maximum number of matrix codes within a set is given by Johnson bounds [133]. It motivates to explore the algorithms generating a set of matrix codes with maximum code set size with codes of any code length ' n ', code weight ' w ' and given maximum auto-correlation and cross-correlation constraint of codes within a set. It also motivates to explore a general scheme generating all such multiple sets of these matrix codes with maximum code set size. Based on these motivations, some objectives are decided to be resolved in this thesis as follows.



1. Exploration to algorithm or scheme generating the multiple sets of one dimensional unipolar (optical) orthogonal codes with maximum code set size for any code length ' n ', code weight ' w ', maximum auto-correlation and cross-correlation constraint of codes within each set.
2. Exploration to algorithm or scheme generating the multiple sets of two dimensional unipolar (optical) orthogonal codes with maximum code set size for any code matrix $L \times N$, code weight ' w ', maximum auto-correlation and cross-correlation constraint of matrix codes within each set.

In this thesis, the topic of deliberation is design of one dimensional as well as two dimensional unipolar (optical) orthogonal codes and their maximal clique sets by proposed general algorithms. The designed one dimensional or two dimensional unipolar (optical) orthogonal codes (OOC) are utilized for assignment of orthogonal codes to all pairs of transmitter of information source and receiver of information sink in the network. The algorithms to design one dimensional unipolar (optical) orthogonal codes and their multiple sets are being compared with already proposed schemes in the literatures for designing one dimensional orthogonal codes. An ideal scheme designing all possible sets of one dimensional unipolar orthogonal codes with maximum cardinality is assumed and compared with the proposed schemes as well as schemes in literature for relative performance evaluation. The algorithms to design two dimensional unipolar (optical) orthogonal codes and their multiple sets are being compared with already proposed schemes in the literature for designing two dimensional orthogonal codes. An ideal scheme designing all possible sets of two dimensional unipolar orthogonal codes with maximum cardinality is assumed and compared with the proposed schemes as well as schemes in literature for relative performance evaluation.

This thesis is organized into six chapters. First chapter gives the historical perspective of optical code division multiple access (OCDMA) and optical cdma codes or unipolar (optical) orthogonal codes. This historical perspective is organized into two subsections. First subsection gives the evolution of one dimensional unipolar (optical) orthogonal codes and optical cdma employing one dimensional orthogonal codes with fixed length and weight. The subsection also deals with the development of multi-length, multi-weight unipolar (optical) orthogonal codes and optical cdma employing these codes. The second subsection deals with the evolution of two dimensional unipolar (optical) orthogonal codes and optical cdma employing these codes. This subsection also gives the development of three and multi-dimensional unipolar (optical) orthogonal codes and optical cdma employing these codes. This chapter also deals with types of optical CDMA based on optical coding as well as multiple access interference with its reduction schemes. The first chapter also addresses motivation and the research problem to be resolved.

Second chapter discusses one dimensional optical orthogonal codes, their conventional representations, the conventional methods to calculate auto-correlation and cross-correlation constraints along-with the properties of sets of codes and the schemes proposed in literature finding code words. This chapter also introduces the cardinality bounds on the set of one dimensional optical orthogonal codes called Johnson's bound. The comparison of these schemes with each other and with an ideal one have also been discussed. The comparison of the scheme with the ideal one gives the idea of further improvements.



In the chapter three, the generation of one dimensional unipolar (optical) orthogonal codes in multiple sets is discussed. Each set contains the codes with maximum cardinality for given code length ' n ', given code weight ' w ', auto-correlation constraints less than or equal to λ_a , and cross-correlation constraints less than or equal to λ_c with positive integer values and boundaries like $1 \leq \lambda_a, \lambda_c < w < n$ and w is co-prime with n . The maximum cardinality or upper bound of each set of codes is given by Johnson bounds. A unique representation named by difference of positions representation (DoPR) and new simple methods for calculation of auto-correlation as well as cross-correlation constraints of one dimensional unipolar (optical) orthogonal codes are also proposed in this chapter. Two search algorithms are proposed which find multiple sets of unipolar (optical) orthogonal codes. The first algorithm finds all possible sets of unipolar (optical) orthogonal codes with maximum cardinality for code length ' n ', code weight ' w ' such that w and n are co-prime, auto-correlation constraint and cross-correlation constraint in the range lying from 1 to $w-1$ using direct search method. This algorithm works well upto $n=47$ and $w=4$ for auto-correlation and cross-correlation constraints lying from 1 to 3. The second algorithm uses clique search method to find all sets of codes not only for the same length and the same weight but also for the multi-length and multi-weight one dimensional unipolar orthogonal codes. This algorithm work well upto $n=256$ and $w=5$ for auto-correlation and cross-correlation constraints lying from 1 to 2. The algorithm work well is quoted in the sense of timing required in execution of programs.

Second algorithm is proposing the codes and their all multiple sets using clique search method which reduces computational complexity. These algorithms are generating their codes in difference of positions representation (DoPR) proposed here. These codes can be converted into proper binary sequences which can be assigned to multiple users of incoherent optical cdma system.

Fourth chapter gives details of two dimensional optical orthogonal codes used in optical CDMA systems. It describes the conventional representations and conventional methods to calculate correlation constraints. It explains the proposed schemes in literature for the design of set of two dimensional optical orthogonal codes. The Johnson's bound or cardinality for the set of two dimensional optical orthogonal codes has also been given here. The ideal scheme for design of two dimensional optical orthogonal codes has been assumed with ideal results and compared with the proposed schemes in literature. This comparison provides an idea about how close the existing schemes are to the ideal one.

Fifth chapter discusses two dimensional unipolar (optical) orthogonal codes, with a new and unique representation of two dimensional optical orthogonal codes, a novel and simple method for calculation of correlation constraints. Two new search algorithms for design of two dimensional unipolar (optical) orthogonal codes through one dimensional unipolar (optical) orthogonal codes and finding their multiple sets have been discussed. The cardinality of each code-set approach the Johnson's bound for different correlation constraints. This newly proposed scheme has also been compared with ideal one which is assumed in chapter four. The first algorithm finds all possible sets of unipolar (optical) orthogonal codes with maximum cardinality for matrix code dimension $(L \times N)$, code weight ' w ' such that w and LN are co-prime, auto-



correlation constraint and cross-correlation constraint from the range 1 to $w-1$ using direct search method. This algorithm works well upto $LN = 46$ and $w=4$ for auto-correlation and cross-correlation constraints lying from 1 to 3. The second algorithm uses clique search method to find all sets of codes not only for the same length and the same weight but also for the multi-length and multi-weight one dimensional unipolar orthogonal codes. This algorithm work well upto $LN= 256$ and $w=5$ for auto-correlation and cross-correlation constraints lying from 1 to 2.

Finally, in chapter six the first conclusion has drawn from the comparison of proposed one dimensional unipolar (optical) orthogonal codes with already proposed schemes to design one dimensional optical orthogonal codes and one assumed scheme with ideal results for one dimensional optical orthogonal codes. The proposed schemes of designing one dimensional optical orthogonal codes is very close to ideal one but with higher computational complexity. The second conclusion drawn from the comparison of proposed two dimensional unipolar (optical) orthogonal with already proposed schemes to design two dimensional optical orthogonal codes and one assumed scheme with ideal results for two dimensional optical orthogonal codes. The proposed scheme of designing two dimensional optical orthogonal codes is very close to ideal one but with higher computational complexity. The third conclusion drawn from comparison of proposed two dimensional unipolar (optical) orthogonal codes with proposed one dimensional unipolar (optical) orthogonal. The cardinality of the set of two dimensional optical orthogonal codes is much better than the set of one dimensional optical orthogonal codes of same temporal length and code parameters at the cost of computational complexity. The design of three dimensional and multidimensional optical orthogonal codes may be taken as future work. The challenge is to reduce the computational complexity of the schemes.

The designed one dimensional unipolar (optical) orthogonal codes can be utilized for direct sequence incoherent optical CDMA system to access the optical fiber in asynchronous manner by multiple users. The designed two dimensional unipolar (optical) orthogonal codes can be utilized for wavelength hopping time spreading optical CDMA system with increased cardinality and spectral efficiency. The multiple sets of these codes are designed. It provides flexibility for selection of set of unipolar orthogonal codes with maximum cardinality. The code set with maximum cardinality provides flexibility for selection of unipolar orthogonal codes from same set.



CHAPTER 2

2. ONE DIMENSIONAL UNIPOLAR (OPTICAL) ORTHOGONAL CODES (1-D U(O)OC)

2.1 Introduction :

The one dimensional unipolar orthogonal codes or optical orthogonal codes or pseudo orthogonal codes are employed with incoherent optical CDMA system. These codes are used for spread spectrum modulation of information bits from every users of optical CDMA system. The same set of these codes is employed at receiver section to demodulate every user's information. To increase the cardinality of optical CDMA system, the set of codes with maximum size is used for the purpose of modulation and demodulation. The one dimensional optical orthogonal code is a binary sequence of code length ' n ' and code weight ' w ' such that ($w < n$). For the codes within set, the maximum non-zero shift auto-correlation and cross-correlation of any pair of codes should be minimum for good orthogonal codes described in this chapter. Conventionally these codes are represented as binary sequences as well as weighted position representation described in next section. The conventional methods of calculating the auto-correlation constraints and cross-correlation constraints are also described in next sections. The already proposed schemes to design the sets of orthogonal codes are also discussed and compared with an assumed ideal scheme.

2.2 Conventional Representations of One Dimensional Unipolar (Optical) Orthogonal Codes

2.2.1 Weighted Position Representation (WPR):

The one dimensional unipolar (optical) orthogonal code word X of code length n and code weight w includes w number of 1's and $(n-w)$ number of 0's. There are n positions of code X in binary form which are termed as 0^{th} position to $(n-1)^{\text{th}}$ position out of which there are w weighted positions and $n-w$ non weighted positions. The code X can be represented by showing only weighted positions of code X . There can be such n representations for each of n circular shifted versions of code X . This type of representation of an unipolar orthogonal codeword may be called as weighted positions representation (WPR) or bit 1's positions representation. For example, suppose one dimensional unipolar orthogonal code X of length $n=19$, code weight $w=4$ such that $X = 1000100001000000100$, which can be represented as WPR (0,4,9,16). Each of n circular shifted versions of code X represent to same unipolar orthogonal code X . All other weighted position representations of code X can be given as (3,8,15,18), (2,7,14,17), (1,6,13,16), (0,5,12,15), (4,11,14,18), (3,10,13,17), (2,9,12,16), (1,8,11,15), (0,7,10,14), (6,9,13,18), (5,8,12,17), (4,7,11,16), (3,6,10,15), (2,5,9,14), (1,4,8,13), (0,3,7,12), (2,6,11,18), (1,5,10,17). Anyone of these can be used to represent the one dimensional unipolar orthogonal code X supposed as above in WPR.



2.2.2 Fixed Weighted Position Representation (FWPR)

The ‘ n ’ representations of a unipolar code in WPR can be reduced by making a compulsory position of bit ‘1’ at position zero. This will reduce the number of weighted positions representations of the unipolar orthogonal code to w from n representations. This reduced weighted positions representation may be called as fixed weighted positions representation (FWPR). The code X in FWPR can be given as $X_{F0} = (f_{x00}, f_{x01}, \dots, f_{x0(w-1)})$ which means that the positions $f_{x00}, f_{x01}, \dots, f_{x0(w-1)}$ are ‘1’ (weighted) while other ‘ $n-w$ ’ positions are ‘0’ (non-weighted). The shifting of X in binary form by $f_{x00}, f_{x01}, \dots, f_{x0(w-1)}$ units in left circularly convert the code X into other FWPRs like $X_{F1}, X_{F2}, \dots, X_{F(w-1)}$.

$$X_{F1} = (f_{x10}, f_{x11}, \dots, f_{x1(w-1)})$$

$$X_{F0} = (f_{x20}, f_{x21}, \dots, f_{x2(w-1)})$$

...

$$X_{F(w-1)} = (f_{x(w-1)0}, f_{x(w-1)1}, \dots, f_{x(w-1)(w-1)})$$

$$X_F = \begin{bmatrix} X_{F0} \\ X_{F1} \\ \vdots \\ X_{F(w-1)} \end{bmatrix} = \begin{bmatrix} f_{x00} & f_{x01} & \dots & f_{x0(w-1)} \\ f_{x10} & f_{x11} & \dots & f_{x1(w-1)} \\ \vdots & \vdots & \ddots & \vdots \\ f_{x(w-1)0} & f_{x(w-1)1} & \dots & f_{x(w-1)(w-1)} \end{bmatrix}$$

The code X in its matrix FWPR X_F contains all FWPR ($X_{F0}, X_{F1}, X_{F2}, \dots, X_{F(w-1)}$) of code X in the rows of matrix FWPR X_F . These rows of X_F always have at least one common element weighted at zero position so that the first column of code matrix X_F is always zero. For the same example as for WPR, $X = 1000100001000000100$, the fixed weighted position representations of code are given as WPR with 0th weighted positions like (0,4,9,16), (0,5,12,15), (0,7,10,14), (0,3,7,12).

$$X_{F0} = (0, 4, 9, 16)$$

$$X_{F1} = (0, 5, 12, 15)$$

$$X_{F2} = (0, 7, 10, 14)$$

$$X_{F3} = (0, 3, 7, 12)$$



The matrix FWPR for this code X is given as $X_F = \begin{bmatrix} 0 & 4 & 9 & 16 \\ 0 & 5 & 12 & 15 \\ 0 & 7 & 10 & 14 \\ 0 & 3 & 7 & 12 \end{bmatrix}$.

Such FWPR representation of an unipolar orthogonal code is not unique as it has w representations of an orthogonal code. To make the representation of an orthogonal code as unique, a new representation is proposed in next chapter.

2.3 Conventional Methods for Calculations of Correlation Constraints of One Dimensional Unipolar (Optical) Orthogonal Codes:

Let two uni-polar code words X and Y belong to a code set with code parameters $(n, w, \lambda_a, \lambda_c)$. $X = (x_0, x_1, \dots, x_{n-1})$, $Y = (y_0, y_1, \dots, y_{n-1})$; $\forall x_t, \forall y_t \in (0,1) \forall t$.

Definition 2.3.1: [25]

The maximum of non-zero shift auto-correlation of uni-polar or binary code X is given as λ_{ax} .

$$\lambda_{ax} \geq \sum_{t=0}^{n-1} x_t x_{t \oplus m} \text{ for } 0 < m \leq n-1.$$

$t \oplus m$ implies $(t+m) \bmod(n)$.

Example 2.3.1(a):

Let the code X with length ' n '=13 and code weight ' w '=4, be [0 1 0 1 0 0 1 0 0 0 1 0 0]. For $0 < m \leq 12$, the left circular shifted binary sequences (X_1, X_2, \dots, X_{12}) of the code X , are as follows.

$X = [0 1 0 1 0 0 1 0 0 0 1 0 0]$, $X_1 = [1 0 1 0 0 1 0 0 0 1 0 0 0]$, $X_2 = [0 1 0 0 1 0 0 0 1 0 0 0 1]$, \dots , $X_{12} = [0 0 1 0 1 0 0 1 0 0 0 1 0]$.

The overlapping of weighted bits or non zero shift auto-correlation of code X with its circular shifted binary sequences (X_1, X_2, \dots, X_{12}) are (0,1,1,2,1,1,1,1,1,1,0). The maximum of all such values is termed as maximum non-zero shift auto-correlation λ_{ax} of the code. It will be 2 in this case.

Definition 2.3.2: [25]

If X_p is weighted positions representation (WPR) [155] of uni-polar orthogonal code X of length ' n ' and weight ' w ', the maximum non-zero shift auto-correlation λ_{ax} of the code is given as $\lambda_{ax} \geq (a + X_p) \cap (b + X_p)$, $(a \neq b)$, $0 \leq (a, b) \leq n-1$. X_p contains ' w ' integer values showing weighted positions or positions of bit 1's of the code X . Here $a + X_p = \{(a + x_p) \bmod n : x_p \in X_p\}$.



Example 2.3.2(a):

Let the uni-polar code $X = [1\ 0\ 1\ 0\ 0\ 1\ 0\ 0\ 0\ 1\ 0\ 0\ 0]$ with code-length ' n '=13, and the code-weight ' w '=4, has its weighted positions representation $X_p = (0,2,5,9)$. The circular shifted sequences of the code X , or $(a+X_p)$ or $(b+X_p)$ for $0 \leq (a,b) \leq 12$, are given as following. $[(0,2,5,9), (1,3,6,10), (2,4,7,11), (3,5,8,12), [(0,4,6,9), (1,5,7,10), (2,6,8,11), (3,7,9,12), (0,4,8,10), (1,5,9,11), (2,6,10,12), (0,3,7,11), (1,4,8,12)]$. The intersection of these circular shifted weighted position sequences $(a+X_p)$ with $(b+X_p)$ is not greater than 2. Hence the maximum non-zero shift auto-correlation of the code X is equal to 2.

Definition 2.3.3:[142]

If X_p is weighted positions representation [155] of uni-polar orthogonal code X of length ' n ' and weight ' w ', the maximum non-zero shift auto-correlation λ_{ax} of the code is also given as $\lambda_{ax} \geq (X_p) \cap (a+X_p)$, $(0 < a \leq n-1)$

Example 2.3.3(a):

Let us take same code X as in examples 2.3.1(a) and 2.3.2(a). The intersection of WPR of code X , $X_p = (0,2,5,9)$ with circular shifted sequences of X or $(a+X_p)$ is not greater than 2. Hence the maximum non-zero shift auto-correlation λ_{ax} of the code X is equal to 2.

Definition 2.3.4: [23],[25]

The auto-correlation constraint λ_a for the set of 1-DUOC (one dimensional unipolar orthogonal codes) is always greater than or equal to maximum non-zero shift auto-correlation λ_{ax} of every code within the set. $\lambda_a \geq \lambda_{ax}$.

Example 2.3.4(a):

Let the set of one dimensional uni-polar orthogonal codes is (X,Y,Z,A,B) . The maximum non-zero shift auto-correlation λ_{ax} of the codes X,Y,Z,A,B are 1,2,1,2,2 respectively. The auto-correlation constraint λ_a for the set is maximum of (1,2,1,2,2) i.e., $\lambda_a = 2$.

Lemma 2.3.5: [23],[25]

For code X , the maximum non-zero shift auto-correlation λ_{ax} satisfy the following relation, $1 \leq \lambda_{ax} \leq w-1$, for 1-DUOC with code parameters $(n, w \geq 2)$.

Proof:

In the uni-polar code with $w \geq 2$, at least one weighted bit will always overlap with one of the $(n-1)$ non-zero circular shifted versions. No uni-polar code with its every non-zero circular shifted version results in ' w ' overlapped weighted bits. Because ' w ' overlapping weighted bits occurs only with the codes un-shifted or zero (mod (n)) circular shifted versions. Then the maximum overlapping of code with its non-zero circular shifted versions is less than w i.e. less than equal to $(w-1)$. Hence for the code parameters $(n, w \geq 2)$ the values of maximum non-zero shift auto-correlation of the codes lies in the range 1 to $(w-1)$. \square



Definition 2.3.6: [23],[25]

The maximum cross-correlation of a uni-polar code X with another code Y and all the $(n-1)$ circular shifted versions of code Y is defined as cross-correlation λ_{cxy} for the pair of codes X and Y and satisfies

$$\lambda_{cxy} \geq \sum_{t=0}^{n-1} x_t y_{t \oplus m} \text{ or } \sum_{t=0}^{n-1} y_t x_{t \oplus m}, \text{ for } 0 \leq m \leq n-1.$$

Example 2.3.6(a):

Let the code length ' n '=13, code weight ' w '=4, the uni-polar code X=[0 1 0 1 0 0 1 0 0 0 1 0 0] and code Y=[1 1 0 1 0 0 0 0 0 1 0 0 0]. The maximum non-zero shift auto-correlation of both X and Y is 2. The overlapping of weighted bits of code Y with X and all 12 circular shifted versions of code X i.e. $(X_1, X_2, \dots, X_{12})$ (as given in example 2.3.1(a)) are (2,0,1,2,1,0,2,1,2,0,2,1,2). The maximum of these cross correlation values is 2 which is the cross-correlation for the pair of codes X and Y, i.e. $\lambda_{cxy} = 2$.

Definition 2.3.7: [25]

If X_p and Y_p are weighted positions representation (WPR) [144] of uni-polar orthogonal code X and Y respectively with code-length ' n ' and weight ' w ', the cross-correlation

$$\lambda_{cxy} \text{ of the pair of code X and Y is given as } \lambda_{cxy} \geq (a + X_p) \cap (b + Y_p), \quad 0 \leq (a, b) \leq n-1.$$

Example 2.3.7(a):

Let the code length ' n '=13, code weight ' w '=4, the uni-polar code X= [1 0 1 0 0 1 0 0 0 1 0 0 0] and code Y= [1 1 0 1 0 0 0 0 0 1 0 0 0] with its weighted positions representation $X_p = (0,2,5,9)$ and $Y_p = (0,1,3,9)$ respectively. The circular shifted sequences of the code X, or $(a+X_p)$ with its weighted positions are given as [(0,2,5,9), (1,3,6,10), (2,4,7,11), (3,5,8,12)], (0,4,6,9), (1,5,7,10), (2,6,8,11), (3,7,9,12), (0,4,8,10), (1,5,9,11), (2,6,10,12), (0,3,7,11), (1,4,8,12)]. The circular shifted sequences of the code Y, or $(b+Y_p)$ with its weighted positions are given as [(0,1,3,9), (1,2,4,10), (2,3,5,11), (3,4,6,12), (0,4,5,7), (1,5,6,8), (2,6,7,9), (3,7,8,10), (4,8,9,11), (5,9,10,12), (0,6,10,11), (1,7,11,12), (0,2,8,12)]. The intersection of these circular shifted sequences $(a+X_p)$ and $(b+Y_p)$ with its weighted positions is not greater than 2. Hence the cross-correlation λ_{cxy} for the code X and Y is equal to 2.

Definition 2.3.8: [153]

If uni-polar code X and Y of length ' n ' and weight ' w ' are represented with its ' w ' weighted positions, the cross-correlation λ_{cxy} of the code is also given as

$$\lambda_{cxy} \geq (X_p) \cap (a + Y_p), \quad (0 \leq a \leq n-1)$$

Alternatively

$$\lambda_{cxy} \geq (Y_p) \cap (a + X_p), \quad (0 \leq a \leq n-1)$$



Example 2.3.8(a):

Let the code length be ' n '=13, code weight ' w '=4, the uni-polar code $X = [1\ 0\ 1\ 0\ 0\ 1\ 0\ 0\ 0\ 1\ 0\ 0\ 0]$ and code $Y = [1\ 1\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 0]$ with their weighted positions representation $X_P = (0,2,5,9)$ and $Y_P = (0,1,3,9)$ respectively. The circular shifted sequences of the code Y , or $(a+Y_P)$ are given as in example 2.3.7(a). The intersection of code X_P and the circular shifted sequences $(a+Y_P)$ with its weighted positions is not greater than 2. Hence the cross-correlation λ_{cxy} of the code X and code Y is equal to 2.

Definition 2.3.9: [23],[25]

The cross-correlation constraint λ_c for the set of 1-DUOCs is always greater than or equal to cross-correlation λ_{cxy} of any pair of codes within the set. $\lambda_c \geq \lambda_{cxy}; \forall x, y$.

Example 2.3.9(a):

Let the set of 1-DUOCs be (X,Y,Z,A,B) . The pairs of codes within set are $(XY,XZ,XA,XB,YZ,YA,YB,ZA,ZB,AB)$. Let the cross-correlation values for these pairs of codes are $(2,1,2,2,1,1,2,1,1,2)$ respectively. The cross-correlation constraint λ_c for the set is maximum of $(2,1,2,2,1,1,2,1,1,2)$, i.e. $\lambda_c = 2$ for the set of codes (X,Y,Z,A,B) .

Lemma 2.3.10: [23],[25]

For the pair of 1-DUOC with code- parameters $(n, w \geq 2)$, X and Y , the cross-correlation λ_{cxy} satisfies the following relation, $1 \leq \lambda_{cxy} \leq w-1$.

Proof: In a pair of uni-polar codes with code parameters $(n, w \geq 2)$, at least one weighted bit of one uni-polar code will always overlapped with other code or one of the $(n-1)$ non-zero circular shifted versions of other code. Further no uni-polar code will results in ' w ' overlapping of weighted bits with other code or non-zero circular shifted versions of other code. Because ' w ' overlapping of weighted bits occurs only with its own un-shifted or zero (mod (n)) circular shifted versions. Thus the maximum overlapping of code with other code or non-zero circular shifted versions of other code may result in less than w or less than equal to $(w-1)$ overlapping. Hence, for the code parameters $(n, w \geq 2)$, the cross-correlation of the pair of codes lies between 1 to $(w-1)$. The one-dimensional uni-polar orthogonal codes with $\lambda_{cxy} = 1$ are perfect uni-polar orthogonal codes, while the codes with $1 < \lambda_{cxy} \leq (w-1)$ are quasi orthogonal. \square

Theorem 2.3.11:

The orthogonality and cardinality of the maximal set of one-dimensional uni-polar orthogonal codes are inversely proportional to each other.

Proof:

The pair of uni-polar codes with $\lambda_c = 1$, is termed as maximum orthogonal



1-DUOC pair. While the pair of uni-polar codes with $\lambda_c = w-1$, is termed as minimum orthogonal pair of 1-DUOC. For $\lambda_a = \lambda_c = \lambda$ where $1 \leq \lambda \leq w-1$, the maximum number of one dimensional uni-polar orthogonal codes Z , within a set, is given by following Johnson

bound [25],[34],[78],[122] $Z(n, w, \lambda) \leq \left\lfloor \frac{1}{w} \left\lfloor \frac{n-1}{w-1} \dots \left\lfloor \frac{n-\lambda}{w-\lambda} \right\rfloor \right\rfloor \right\rfloor = J_A(n, w, \lambda)$. Here $\lfloor a \rfloor$ represents largest integer less than equal to a . For $\lambda = w-1$, $Z(n, w, w-1) \leq \left\lfloor \frac{1}{n} {}^nC_w \right\rfloor = \left\lfloor \frac{1}{w} \left\lfloor \frac{n-1}{w-1} \dots \left\lfloor \frac{n-(w-1)}{1} \right\rfloor \right\rfloor \right\rfloor$ which represent maximum number of 1-

DUOCs within one set with minimum orthogonality. For, $\lambda = 1$, $Z(n, w, 1) \leq \left\lfloor \frac{1}{w} \left\lfloor \frac{n-1}{w-1} \right\rfloor \right\rfloor$, which

represents to minimum number of uni-polar orthogonal codes in one set with maximum orthogonality. For $(\lambda = p)$, $(1 < p < w-1)$, the cardinality of maximal set is

$Z(n, w, p) \leq \left\lfloor \frac{1}{w} \left\lfloor \frac{n-1}{w-1} \dots \left\lfloor \frac{n-p}{w-p} \right\rfloor \right\rfloor \right\rfloor$, which is less than the cardinality of maximal set for $(\lambda = p+1)$,

$Z(n, w, p+1) \leq \left\lfloor \frac{1}{w} \left\lfloor \frac{n-1}{w-1} \dots \left\lfloor \frac{n-p}{w-p} \left\lfloor \frac{n-(p+1)}{w-(p+1)} \right\rfloor \right\rfloor \right\rfloor \right\rfloor$. While the orthogonality for the set with $(\lambda = p)$ is

greater than for the set with $(\lambda = p+1)$. It proves that orthogonality and cardinality of maximal set are inversely related to each other. \square

Lemma 2.3.12:

The maximal set of 1-DUOCs with parameters $(n, w, \lambda_a, \lambda_c)$ forms a maximal clique of codes.

Proof:

All the codes in a set are such that every pair of codes is having correlation properties within given range. If the codes are assumed to be nodes, then each node is connected with all others with the given properties. This shows that all the codes within set form a clique. If the cardinality of the set is maximum or equal to upper bound; it means that the formed clique of codes is maximal. A code is chosen and we can keep on adding another code to extend the set so that extended set is a clique. Once it is no more possible to extend the set further, we have achieved a maximal clique. \square

Theorem 2.3.13:

For the code parameters $(n, w, \lambda_a, \lambda_c)$, the cardinality of maximal clique set and number of maximal clique sets are inversely proportional to each other.

Proof: As per Theorem 2.1, a single set of the 1-DUOC is possible with minimum orthogonality or $(\lambda = w-1)$ and maximum cardinality. Moreover, for $(\lambda = p)$, $(1 \leq p < w-1)$, the cardinality of the maximal set is less than for $(\lambda = p+1)$. Then more codes are available for forming more sets for $(\lambda = p)$ than for $(\lambda = p+1)$. It proves that cardinality of maximal set and numbers of



maximal sets are inversely proportional to each other. \square

Lemma 2.3.14:

The minimum cross-correlation among the multiple maximal clique sets for the code parameters $(n, w, \lambda_a, \lambda_c)$ is equal to $(\lambda_c + 1)$.

Proof:

For the code parameters $(n, w, \lambda_a, \lambda_c)$, the maximal clique set contains the codes with auto-correlation constraint less than or equal to λ_a and cross-correlation constraint less than or equal to λ_c . The cross correlation between two independent maximal clique sets is equal to maximum cross-correlation for the pair of codes. One code is taken from one set and other one from the second. This maximum cross-correlation cannot be less than or equal to λ_c because both the sets are maximal. It will always be greater than λ_c . Hence the minimum value of the cross-correlation among the multiple independent maximal clique sets is equal to $(\lambda_c + 1)$. This also implies that no code shall be common between two maximal clique sets. If such a code exist, cross correlation between codes taken from two sets will be less than equal to λ_c and thus sets are not maximal clique sets. \square

Lemma 2.3.15: [25],[34],[78],[122]

For $\lambda_a = \lambda_c = \lambda$ where $1 \leq \lambda \leq w-1$, The maximum number of unipolar/optical orthogonal codes, Z, in one set is given by the following Johnson bounds. Johnson's bound A is,

$$Z(n, w, \lambda) \leq \left\lfloor \frac{1}{w} \left\lfloor \frac{n-1}{w-1} \dots \dots \left\lfloor \frac{n-\lambda}{w-\lambda} \right\rfloor \right\rfloor \right\rfloor = J_A(n, w, \lambda).$$

The Johnson's bound B holds only when $w^2 > n\lambda$, and is,

$$Z(n, w, \lambda) \leq \text{Min} \left(1, \left\lfloor \frac{w-\lambda}{w^2 - n\lambda} \right\rfloor \right) = J_B(n, w, \lambda);$$

The improved Johnson's Bound C for any integer k, $1 \leq k \leq \lambda-1$; such that $(w-k)^2 > (n-k)(\lambda-k)$, is given as

$$Z(n, w, \lambda) \leq \left\lfloor \frac{1}{w} \left\lfloor \frac{n-1}{w-1} \dots \dots \left\lfloor \frac{n-(k-1)}{w-(k-1)} h \right\rfloor \right\rfloor \right\rfloor = J_C(n, w, \lambda);$$



$$\text{Here } h = \min \left((n-k), \left\lfloor \frac{(n-k)(w-\lambda)}{(w-k)^2 - (n-k)(\lambda-k)} \right\rfloor \right) ;$$

$\lfloor a \rfloor$ denotes the largest integer value less than 'a'. In short, J_A , J_B , and J_C denotes the Johnson bound A, B, and C respectively [25], [45], [133].

The one dimensional unipolar orthogonal codes have application in incoherent optical code division multiple access systems. The optical orthogonal pulsed signal can be generated by putting an optical pulse at the each position of bit '1' and no pulses at the positions of bit '0's in the unipolar orthogonal codes using optical delay lines [12,18]. In literature there is a lot of one dimensional optical orthogonal code design schemes are proposed [8-11, 14-14, 21, 25, 55-57, 59, 64, 65, 68, 77-78, 90-91, 95-96, 103-104, 112, 125, 133, 138, 140, 145]. Out of which some schemes are discussed which design set of unipolar orthogonal codes for fixed values of code length, code weight, and correlation constraints.

2.4 Already Proposed 1-D OOC Design Schemes in Literature

In literature some optical orthogonal code design schemes are proposed time to time, some of these are discussed as following

2.4.1 OOCs based on Prime sequences

Suppose a Galois Field $GF(p) = (0, 1, 2, \dots, p-1)$, p is a prime, is used to construct the prime sequence $S_x^p = \{s_x^p(0), s_x^p(1), s_x^p(2), \dots, s_x^p(p-1)\}$, here $s_x^p(j) = x \cdot j \pmod{p}$, for $x, j \in GF(p)$, The binary code word $C_x^p = \{c_x^p(0), c_x^p(1), \dots, c_x^p(p-1)\}$ with

$$c_x^p(i) = \begin{cases} 1 & \text{for } i = s_x^p(j) + jp \pmod{p}; j = 0, 1, \dots, p-1 \\ 0 & \text{otherwise} \end{cases} \quad \text{Where } i = 0, 1, \dots, p^2-1.$$

This scheme generate the set of optical orthogonal codes $(n, w, \lambda_a, \lambda_c)$ for any prime number p , such that weight $w = p$, length $n = p^2$, auto-correlation constraint $\lambda_a = p-1$ and cross correlation constraint $\lambda_c = 2$. The number of optical orthogonal codes in the set are given by $N = p$ [15]. The computational complexity of this scheme is of the order $O(w^3)$.

For example $p=5$, $GF(p) = (0, 1, 2, 3, 4)$, $S_0^p = \{0, 0, 0, 0, 0\}$, $S_1^p = \{0, 1, 2, 3, 4\}$, $S_2^p = \{0, 2, 4, 1, 3\}$, $S_3^p = \{0, 3, 1, 4, 2\}$, $S_4^p = \{0, 4, 3, 2, 1\}$.

$$C_0^p = \{10000 \ 10000 \ 10000 \ 10000 \ 10000\}$$

$$C_1^p = \{10000 \ 01000 \ 00100 \ 00010 \ 00001\}$$

$$C_2^p = \{10000 \ 00100 \ 00001 \ 01000 \ 00010\}$$



$$C_3^P = \{10000\ 00010\ 01000\ 00001\ 00100\}$$

$$C_4^P = \{10000\ 00001\ 00010\ 00100\ 01000\}$$

These designed codes are assigned to users of incoherent optical CDMA system to improve the bit error rate (BER) performance of the system [16, 69].

2.4.2 Quasi Prime OOCs

This scheme is the extension of the OOC set based on prime sequences and is explained in [55]. A quasi prime code C_{xk}^{qp} is a time shifted and extended (or contracted) version of prime sequence code C_x^P . It is given as, with q number of 1's, $c_{xk}^{qp}(i) = c_x^P([i + kp]_n)$; where $i = 0, 1, \dots, qp-1$. Here in code set $(n, w, \lambda_a, \lambda_c)$ $n = qp$, $(r-1)p < q < rp$; p is a prime number, q, r and k are positive integers; weight $w = q$; auto-correlation constraint $\lambda_a = (p-1)r$, cross-correlation constraint $\lambda_c = 2$ and the number of code-words $N = p$; The computational complexity of this scheme is of the order of $O(w^3)$. for example $p = 5$, $q = 7$, $k = 3$, with the example given as in (2.4.1), the extended version of prime codes

$$C_{0k}^{qp} = \{10000\ 10000\ 10000\ 10000\ 10000\ 10000\ 10000\}$$

$$C_{1k}^{qp} = \{00010\ 00001\ 10000\ 01000\ 00100\ 00010\ 00001\}$$

$$C_{2k}^{qp} = \{01000\ 00010\ 10000\ 00100\ 00001\ 01000\ 00010\}$$

$$C_{3k}^{qp} = \{00001\ 00100\ 10000\ 00010\ 01000\ 00001\ 00100\}$$

$$C_{4k}^{qp} = \{00100\ 01000\ 10000\ 00001\ 00010\ 00100\ 01000\}$$

for another example $p = 5$, $q = 4$, $k = 3$, with the example given as in (2.2.1), the contracted version of prime codes

$$C_{0k}^{qp} = \{10000\ 10000\ 10000\ 10000\}$$

$$C_{1k}^{qp} = \{00010\ 00001\ 10000\ 01000\}$$

$$C_{2k}^{qp} = \{01000\ 00010\ 10000\ 00100\}$$

$$C_{3k}^{qp} = \{00001\ 00100\ 10000\ 00010\}$$

$$C_{4k}^{qp} = \{00100\ 01000\ 10000\ 00001\}$$

2.4.3 OOCs based on Quadratic Congruence

The optical orthogonal code $C_x^P = \{c_x^P(0), c_x^P(1), \dots, c_x^P(n-1)\}$ is based on quadratic placement operator $y_x(k)$, for a prime integer p , $x = [1, 2, \dots, p-1]$, and $k = \left\lfloor \frac{i}{p} \right\rfloor$, such that

$$c_x^P(i) = \begin{cases} 1 & \text{if } y_x(k) + kp = i, \\ 0 & \text{otherwise} \end{cases} \quad i = 0 : p^2 - 1$$



Here $y_x(k) = \frac{xk(k+1)}{2} \pmod{p}$ and $y_x(k+1) \equiv [y_x(k) + k + 1] \pmod{p}$

here the orthogonal code set $(n, w, \lambda_a, \lambda_c)$ is constructed for the length $n = p^2$; weight $w = p$; $\lambda_a = 2$; $\lambda_c = 4$ as in [45]. The computational complexity of this scheme and its extended version is of the order $O(w^3)$.

For example, $p=5$, $x=2$, the quadratic placement operators are given as $y_1^k = \{0 \ 1 \ 3 \ 1 \ 0\}$, $y_2^k = \{0 \ 2 \ 1 \ 2 \ 0\}$, $y_3^k = \{0 \ 3 \ 4 \ 3 \ 0\}$, $y_4^k = \{0 \ 4 \ 2 \ 4 \ 0\}$ and corresponding codes are given as

$$C_1^5 = \{10000 \ 01000 \ 00010 \ 01000 \ 10000\}$$

$$C_2^5 = \{10000 \ 00100 \ 01000 \ 00100 \ 10000\}$$

$$C_3^5 = \{10000 \ 00010 \ 00001 \ 00010 \ 10000\}$$

$$C_4^5 = \{10000 \ 00001 \ 00100 \ 00001 \ 10000\}$$

The Extended Quadratic Congruence, where the length of Quadratic Congruence code is extended, can be used for construction of code of length $n = p(2p-1)$, weight $w = p$, autocorrelation constraint $\lambda_a = 1$, and cross correlation constraint $\lambda_c = 2$ as explained in [46].

$$C_1^5 = \{100000000 \ 010000000 \ 000100000 \ 010000000 \ 100000000\}$$

$$C_2^5 = \{100000000 \ 001000000 \ 010000000 \ 001000000 \ 100000000\}$$

$$C_3^5 = \{100000000 \ 000100000 \ 000010000 \ 000100000 \ 100000000\}$$

$$C_4^5 = \{100000000 \ 000010000 \ 001000000 \ 000010000 \ 100000000\}$$

2.4.4 OOCs based on Projective Geometry

A Projective Geometry $PG(m, q)$ of order m , is constructed from a vector space $V(m+1, q)$ of dimension $m+1$ over $GF(q)$, where $GF(q)$ is Galois Field with q elements.. An s -space in a $PG(m, q)$ corresponds to $(s+1)$ dimensional space through the origin in $V(m+1, q)$ [64]. Here one-dimensional subspaces of V are the points and the two dimensional subspaces of V are

the lines. Number of points in $PG(m, q)$, $n = \left(\frac{q^{m+1} - 1}{q - 1} \right)$ will give the length of the codeword

Number of points in the s -space, $w = \left(\frac{q^{s+1} - 1}{q - 1} \right)$ will give the weight of the codeword The

intersection of two s space is an $(s-1)$ -space. Number of points in the $(s-1)$ -space,

$\lambda = \left(\frac{q^s - 1}{q - 1} \right) = \max(\lambda_a, \lambda_c)$. The cyclic shift of an s space is also an s -space. The orbit is the set

of all s -spaces that are cyclic shift of each other. The number of code words is always equal to number of complete orbits. A codeword consists of discrete logarithm of points in each



representative s-space. The intersection of two s space is an (s-1) space. Number of points in the (s-1)-space, $\lambda = \left(\frac{q^s - 1}{q - 1} \right) = \max(\lambda_a, \lambda_c)$.

The cyclic shift of an s space is also an s-space. The orbit is the set of all s-spaces that are cyclic shift of each other. The number of code words is always equal to number of complete orbits. A codeword consists of discrete logarithm of points in each representative s-space.

Total number of s-spaces, $M_s = \frac{\binom{n}{s+1}}{\binom{w}{s+1}}$ Total number of code words constructed using

$PG(m, q)$ for given value of s are equal to $M = \left\lfloor \frac{M_s}{n} \right\rfloor$, [64,65].

For example, $m=2, q=2$, then $n = 7, w = q+1=3$, for $s=1, \lambda=1$, the number of lines $M_1 = n(n-1)/w(w-1) = 7$ for which total number of code words $M = \lfloor M_1/n \rfloor = 1$, using Galois Field $GF(q^{m+1})$ i.e. $GF(8)$ and taking x^3+x^2+1 as the primitive polynomial with primitive element α , such that $\alpha^i = \beta$ to be element of $GF(8)$ for $0 \leq i \leq q^{m+1} - 2$. Here $\alpha^0 = 001, \alpha^1 = 010, \alpha^2 = 100, \alpha^3 = 101, \alpha^4 = 111, \alpha^5 = 011, \alpha^6 = 110$, with the lines of $PG(2,2)$ with their constituent points can be given such as $\{(0,1,5), (0,2,3), (0,4,6), (1,2,6), (1,3,4), (2,4,5), (3,5,6)\}$. These lines represent to same single code with their weighted positions given by any one of the lines as above.

2.4.5 OOCs based on Error Correcting Codes

An 't' error correcting code is represented by (n, d, w) , where n is length, d is minimum hamming distance between any two code words, w is the constant weight of a code from the code-set. The minimum distance $d \geq 2t+1$. An OOC $(n, w, \lambda_a, \lambda_c)$ is equivalent to constant weight error correcting codes with minimum distance $d = 2w - 2\lambda$, where λ is maximum of (λ_a, λ_c) [45,46,54,66]. Only those error correcting code are selected for optical orthogonal code set whose cyclic shifts are also code word. For example the constant weight error correcting codes $(n, d, w) = (19, 4, 3)$ can be used to generate optical orthogonal codes $(n, w, \lambda) = (19, 3, 1)$, here $\lambda = (2w - d)/2$. The generated optical orthogonal codes are $C_1 = (12, 17, 18), C_2 = (11, 15, 18)$, and $C_3 = (8, 16, 18)$.

2.4.6 OOCs based on Hadamard Matrix

The hadamard matrix of lowest order 2 is as given below

$$H_2 = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} \text{ and}$$

The hadamard matrix of order 2n can be given as



$$H_{2n} = \begin{bmatrix} H_n & H_n \\ H_n & \overline{H_n} \end{bmatrix}$$

$\overline{H_n}$ is complement of H_n

The possible order of hadamard matrix is 2,4, 8,16,42, 64,....

The construction of optical orthogonal codes using hadamard matrix can be studied by taking hadamard matrix of order 8 in the given example.

$$H_8 = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \end{bmatrix}$$

the other matrix H_7 can be constructed from Hadamard matrix H_8 by deleting first row and first column, hence H_7 is

$$H_7 = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \end{bmatrix}$$

Each row from H_7 can be written in form of weight set as $R_1 = (1,3,5)$, $R_2 = (0,3,4)$, $R_3 = (2,3,6)$, $R_4 = (1,2,3)$, $R_5 = (1,4,6)$, $R_6 = (0,5,6)$, $R_7 = (2,4,5)$. After checking the periodicity, we find $R_1 = R_5$; $R_2 = R_3$; $R_4 = R_6$; hence non repeated orthogonal codes are $C_1 = 0101010$; $C_2 = 1001100$; $C_3 = 1110000$; $C_4 = 0010110$;

The generated orthogonal code set of length $n = 7$; weight $w = 3$; $\lambda_a = 1$; $\lambda_c = 2$ In the same way any Hadamard matrix of order n can be used to generate the matrix of order $n-1$ by deleting 1st row and 1st column. The rows of the matrix of order $n - 1$ form a code set. From the same code set the repeated or cyclically shifted codes are included only once to form the optical orthogonal code set of length $n = 4t-1$, weight $w = 2t-1$, $\lambda_a = t-1$, $\lambda_c = t$, t is any positive integer [90].



2.4.7 OOCs based on Skolem Sequences

The skolem sequences of order M , can be written as collection of ordered pairs $\{(a_i, b_i) : 1 \leq i \leq M, b_i - a_i = i\}$ with $\bigcup_{i=1}^M \{a_i, b_i\} = \{1, 2, \dots, 2M\}$. The skolem sequence of order M exist only for $M \equiv 0 \pmod{4}$ or $M \equiv 1 \pmod{4}$. M is the number of code words and the length of code word $n = 6M+1$. The orthogonal codes of weight $w = 3$ can be written as $\{x_{i1}, x_{i2}, x_{i3}\}$, x_{ij} represents the j^{th} position of bit '1' in the

i^{th} code word for $1 \leq i \leq M$.

$x_{i1} = 0$ for all i ,

$x_{i2} = i$ for all i ,

x_{i3} is obtained from the skolem sequence in the way such that for example skolem sequence of order $M=5$ is given as $S = \{(1,2) (7,9) (3,6) (4,8) (5,10)\}$ For $i=1$, $x_{i3} = M+2 = 7$;

For $i=2$, $x_{i3} = M+9 = 14$; For $i=3$, $x_{i3} = M+6 = 11$; For $i=4$, $x_{i3} = M+8 = 13$;

For $i=5$ $x_{i3} = M+10 = 15$; The optical orthogonal code set is $\{(0,1,7), (0,2,14), (0,3,11), (0,4,13), (0,5,15)\}$ The corresponding codes of length $n=31$, $w=3$, $\lambda_a = 1$, $\lambda_c = 1$;

$C_1 = (11000001000000000000000000000000)$

$C_2 = (10100000000000010000000000000000)$

$C_3 = (10010000000100000000000000000000)$

$C_4 = (10001000000001000000000000000000)$

$C_5 = (10000100000000010000000000000000)$

Similarly other code words of length $n=6M+1$, weight $w=3$ and $\lambda_a = 1$, $\lambda_c = 1$ can be generated using skolem sequences of order M as in [91].

2.4.8 OOCs based on Table of Prime

The elements of Galois Field $GF(p)$, where p is prime, are $(1, 2, 3, \dots, p-1)$. Suppose α is a primitive root for prime p , then all the elements of $GF(p)$ can be represented by $\{\alpha^x, \text{ for } x = (0, 1, 2, \dots, p-2)\}$. The prime sequence codes are given as $S_1^p = (\alpha^0, \alpha^1, \dots, \alpha^{p-2}) \pmod{p}$;

$S_2^p = 2 S_1^p \pmod{p}$; $S_3^p = 3 S_1^p \pmod{p}$; ... $S_{p-1}^p = (p-1) S_1^p \pmod{p}$.

It can be best understood by following example for $p=5$ and it's primitive root $\alpha = 2$ and $GF(5) = (1, 2, 3, 4)$ then $2^0 = 1$, $2^1 = 2$, $2^2 = 4$, $2^3 = 8 \pmod{5} = 3$. The prime sequence codes $S_1^p = (1, 2, 4, 3)$

$S_2^p = (2, 4, 8, 6) \pmod{5} = (2, 4, 3, 1)$

$S_3^p = (3, 6, 12, 9) \pmod{5} = (3, 1, 2, 4)$

$S_4^p = (4, 8, 16, 12) \pmod{5} = (4, 3, 1, 2)$

Then the orthogonal code set with weighted positions is given below

$C_1^p = (1, p+2, 2p+4, 3p+3) = (1, 7, 14, 18)$



$$C_2^p = (2 \ p+4 \ 2p+3 \ 3p+1) = (2 \ 9 \ 13 \ 16)$$

$$C_3^p = (3 \ p+1 \ 2p+2 \ 3p+4) = (3 \ 6 \ 12 \ 19)$$

$$C_4^p = (4 \ p+3 \ 2p+1 \ 3p+2) = (4 \ 8 \ 11 \ 17)$$

The corresponding $M = p-1 = 4$ codes of length $n = p^2 - p = 20$, weight $w = p-1 = 4$, $\lambda_a = 1$, $\lambda_c = p-2 = 3$ will be

$$C_1 = (01000 \ 00100 \ 00001 \ 00010);$$

$$C_2 = (00100 \ 00001 \ 00010 \ 01000);$$

$$C_3 = (00010 \ 01000 \ 00100 \ 00001);$$

$$C_4 = (00001 \ 00010 \ 01000 \ 00100);$$
 Similarly for other prime p and root

α , the $GF(p)$, its elements, prime sequence codes and then orthogonal code set can be generated as in [91].

2.4.9 OOCs based on Number Theory

This generate [91] the orthogonal codes $(n, 3, 2, 2)$, for length n to be prime $n = 3t+2$ for some selected integer values of t and α is primitive root of n . The i^{th} codeword out of t possible code words is given by following equation

$$C_i = \{1, \alpha^{w+(i-1)}, \alpha^{2w+d+(i-1)}\}, \text{ where } d = \lambda_c$$

It can be best understood by following example of $t=5$, $n=17$, $w=3$, $\lambda_a = \lambda_c = 2$.

The primitive root of prime number 17 is $\alpha=3$.

$$S_1 = (1, \alpha^3, \alpha^8) = (1, 10, 16)$$

$$S_2 = (1, \alpha^4, \alpha^9) = (1, 13, 14)$$

$$S_3 = (1, \alpha^5, \alpha^{10}) = (1, 5, 8)$$

$$S_4 = (1, \alpha^6, \alpha^{11}) = (1, 15, 7)$$

$$S_5 = (1, \alpha^7, \alpha^{12}) = (1, 11, 4)$$

The corresponding codes are

$$C_1 = (01000000001000001)$$

$$C_2 = (01000000000001100)$$

$$C_3 = (01000100100000000)$$

$$C_4 = (01000001000000010)$$

$$C_5 = (01001000000100000).$$

2.4.10 OOCs based on Quadratic Residues

For any prime p , the quadratic residues (QR) a is defined as $a = x^2 \pmod{p}$; for any integer x . The prime ' p ' has following residues $(0, x_1, x_2, \dots, x_{(p-1)/2})$ the QR sequence is $Q_1 = (q_1, q_2, q_3, \dots, q_p)$ with $q_1 = q_p = 0$; $q_2 = q_{p-1} = x_1$; $q_3 = x_2$; and $q_k = q_{p-k+1}$ for $1 \leq k \leq p$. The j^{th} QR sequence is obtained by multiplying Q_1 by j with all elements are given under \pmod{p} for $j = 1$ to $p-1$. These $(p-1)$ QR sequence when considered as weighted positions of the binary codes with length n represents the orthogonal code set.

For example $p=5$, the quadratic residues are $(0, 1, 4)$

$$\text{QR sequence } Q_1 = (0, 1, 4, 1, 0)$$

$$\text{QR sequence } Q_2 = (0, 2, 3, 2, 0)$$



QR sequence $Q_3 = (0, 3, 2, 3, 0)$

QR sequence $Q_4 = (0, 4, 1, 4, 0)$

the orthogonal code set $(n, w, \lambda_a, \lambda_c) = (p^2, p, 2, 2)$

$S_1 = (0, p+1, 2p+4, 3p+1, 4p+0) = (0, 6, 14, 16, 20)$

$S_2 = (0, p+2, 2p+3, 3p+2, 4p+0) = (0, 7, 13, 17, 20)$

$S_3 = (0, p+3, 2p+2, 3p+3, 4p+0) = (0, 8, 12, 18, 20)$

$S_4 = (0, p+4, 2p+1, 3p+4, 4p+0) = (0, 9, 11, 19, 20)$

The corresponding code words of code set $(25, 5, 2, 2)$ are...

$C_1 = (10000\ 01000\ 00001\ 01000\ 10000)$

$C_2 = (10000\ 00100\ 00010\ 00100\ 10000)$

$C_3 = (10000\ 00010\ 00100\ 00010\ 10000)$

$C_4 = (10000\ 00001\ 01000\ 00001\ 10000)$ As generated in [91].

2.4.11 OOCs based on Balanced Incompleted Block Design (BIBD)

In [68], there are two families of OOC sets are constructed using BIBD. First is $(n, w, 1, 1)$ with optimal cardinality N , the second is $(n, w, 1, 2)$ with cardinality $2N$. the code design is described [1,68] as follows.

- **$(n, w, 1, 1)$ OOC for odd w**

The weight $w = 2m+1$ for a positive integer m , the code length $n = w(w-1)t+1$ for those values of t so that n be a prime number. Consider a Galois Field $GF(n)$ with α be the primitive root of $GF(n)$ such that values of $\{ \log_\alpha[\alpha^{2mkt} - 1] \}$ for $1 \leq k \leq m$ are all distinct with modulo m . The generated code $C_i = [\alpha^{mi}, \alpha^{mi+2mt}, \alpha^{mi+4mt}, \dots, \alpha^{mi+4m^2t}]$ for $i = 0$ to $t-1$ such that code set C_a contains $(C_0, C_1, C_2, \dots, C_{t-1})$ optical orthogonal codes.

For example $w=3, t=5$ which shows $m=1, n=31, \alpha=3$ (primitive root of $GF(31)$). The codes are generated as follows

$$C_0 = [1, \alpha^{10}, \alpha^{20}] = [1, 25, 5]$$

$$C_1 = [\alpha, \alpha^{11}, \alpha^{21}] = [3, 13, 15]$$

$$C_2 = [\alpha^2, \alpha^{12}, \alpha^{22}] = [9, 8, 14]$$

$$C_3 = [\alpha^3, \alpha^{13}, \alpha^{23}] = [27, 24, 11]$$

$$C_4 = [\alpha^4, \alpha^{14}, \alpha^{24}] = [19, 10, 2]$$

- **$(n, w, 1, 1)$ OOC for even w**

The weight $w = 2m$ for a positive integer m , the code length $n = w(w-1)t+1$ to be prime for some selected values of t . the Galois Field $GF(n)$ with α be the primitive root of $GF(n)$ such that values of $\{ \log_\alpha[\alpha^{2mkt} - 1] \}$ for $1 \leq k \leq m$ are all distinct with modulo m . The generated code $C_i = [0, \alpha^{mi}, \alpha^{mi+2mt}, \alpha^{mi+4mt}, \dots, \alpha^{mi+4m(m-1)t}]$ for $i = 0$ to $t-1$, such that code set C_a contains $(C_0, C_1, C_2, \dots, C_{t-1})$ optical orthogonal codes.



- **$(n, w, 1, 2)$ OOC**

The code set of $(n, w, 1, 2)$ OOC can be generated from the code set $(n, w, 1, 1)$ OOC as follows, suppose $(n, w, 1, 1)$ OOC code set C_a contains the codes $(C_0, C_1, C_2, \dots, C_{t-1})$. By reversing the order of bit position of all the code, the generated code words are $(C_0, C_1, C_2, \dots, C_{t-1})$. If $C_0 = (c_0 c_1 c_2 c_3 \dots c_{n-1})$; c_j is either 0 or 1 for j to be 0 to $n-1$, then $C_0' = (c_{n-1} c_{n-2} c_{n-3} \dots c_2 c_1 c_0)$, similarly for others. The code set of $(n, w, 1, 2)$ is defined as $C_b = (C_0, C_1, C_2, \dots, C_{t-1}, C_0', C_1', C_2', \dots, C_{t-1}')$.

2.5 Comparisons with Ideal Scheme

All the schemes proposed in literature, for design of one dimensional optical orthogonal codes, have some limitation over cardinality of code set, number of code sets designed, specific code length, code weight, and correlation constraints. It can be imagined an ideal scheme for design of one dimensional unipolar (optical) orthogonal codes and their multiple possible sets for all general values of code length, code weight and correlation constraints. The ideal scheme might be generating one dimensional unipolar (optical) orthogonal codes for any code length $n, n > 0$, any weight $w, 0 < w \leq n$ and correlation constraints λ_a, λ_c , such that $1 \leq \lambda_a, \lambda_c \leq w-1$ along with all possible such sets for given parameters $(n, w, \lambda_a, \lambda_c)$. The computational complexity of ideal scheme should be very low. The detailed comparison is given in following Table 2.1.

2.6 Conclusion

In this chapter, the conventional representation and conventional methods for calculation of auto-correlation constraint and cross correlation constraints are discussed. Besides of it, the properties of one dimensional unipolar (optical) orthogonal codes are described under some lemmas, definitions and theorems which are nothing but the re-explanation of the properties observed in literature for optical orthogonal codes. Some of schemes already proposed in literature, are detailed with design of one dimensional optical orthogonal codes within a set for specific code length ' n ', code weight ' w ' and correlation constraints. These schemes are being compared with an assumed ideal scheme which might be generating all possible sets of one dimensional unipolar (optical) orthogonal codes with maximum cardinality for general values of code length ' n ', code weight ' w ' and correlation constraints (λ_a, λ_c) .

The next chapter deals with difference of positions representation (DoPR) and a new less complex method to calculate correlation constraints of one dimensional unipolar (optical) orthogonal codes. It also discuss about two proposed schemes to design one dimensional unipolar (optical) orthogonal codes and their some new properties by mentioning lemmas, definitions and theorems.



OOc(1D) based on	Code length 'n'	Weight 'w'	Auto- correlation constraint λ_a	Cross- correlation constraint λ_c	Cardina lity of code- set	No. of code sets	Computational complexity	Other comments
Prime Sequence	p^2	p	p-1	2	$p < Z$	one	$O(w^3)$	p is a prime number
Quasi- prime	qp	q	r(p-1)	2r	$P < Z$	one	$O(w^3)$	'', (r-1)p< q<rp r,q +ve intg
Qad. Congr uences	p^2	p	2	4	$p-1 < Z$	one	$O(w^3)$	p is a prime
Projectiv e Geometr y PG(m,q)	$\left(\frac{q^{m+1}-1}{q-1}\right)$	$\left(\frac{q^{s+1}-1}{q-1}\right)$	$\left(\frac{q^s-1}{q-1}\right)$	$\left(\frac{q^s-1}{q-1}\right)$	$=Z$	one	$\ll O(n^w)$	q is no. of elements in Galois field GF(q)
Error correctin g codes	n	w	(2w-d)/2	(2w-d)/2	$=Z$	one	$\ll O(n^w)$	d is min. hamming dist.
Hadamar d Matrix	2^v-1	v	1	2	$< Z$	one	$\ll O(n^w)$	v is +ve integer
Skolem sequence s	6M+1	3	1	1	$M < Z$	one	$\ll O(n^w)$	M is +ve intg.
Table of Prime	p^2-p	p-1	1	p-2	$p-1 < Z$	one	$\ll O(n^w)$	p is a prime
Number Theory	$n=3t+2$	3	2	2	t	one	$\ll O(n^w)$	n is a prime
Quad. residues	p^2	p	2	2	p-1	one	$\ll O(n^w)$	P is a prime
BIBD	$n=w(w-1)t+1$	$w=2m$ $w=2m+1$	1	1,2	t	one	$\ll O(n^w)$	n is a prime, m +ve intg.
Ideal Scheme	$n>0$	$0<w<n$	1 to w-1	1 to w-1	Z	All possible	$\ll O(n^w)$	This scheme is not in existence

Table 2.1: Comparison of already proposed 1-D OOCs design schemes with ideal scheme.



CHAPTER 3

3. DESIGN OF ONE DIMENSIONAL UNIPOLAR (OPTICAL) ORTHOGONAL CODES AND THEIR MAXIMAL CLIQUE SETS

3.1 Introduction:

In this chapter, the generation of one dimensional unipolar (optical) orthogonal codes in multiple sets is discussed. Each set contains the codes with maximum cardinality for given code length ' n ', given code weight ' w ', auto-correlation constraints less than or equal to λ_a , and cross-correlation constraints less than or equal to λ_c with positive integer values and boundaries like $1 \leq \lambda_a, \lambda_c < w < n$ and w is co-prime with n . The maximum cardinality or upper bound of each set of codes is given by Johnson bounds [25],[45],[89],[133]. A unique representation named by difference of positions representation (DoPR) and new lower complex method for calculation of auto-correlation as well as cross-correlation constraints of one dimensional unipolar (optical) orthogonal codes are also proposed in this chapter. These generated codes provide flexibility for selection of one dimensional unipolar (optical) orthogonal codes from same set to multiple users of incoherent optical code division multiple access (CDMA) systems. The generated multiple sets provide flexibility for selection of a set of one dimensional unipolar orthogonal codes to be assigned to a set of users of incoherent optical CDMA systems. Two search algorithms are proposed which find multiple sets of unipolar (optical) orthogonal codes. The first algorithm finds all possible sets of unipolar (optical) orthogonal codes with maximum cardinality for code length ' n ', code weight ' w ' such that w and n are co-prime, auto-correlation constraint and cross-correlation constraint from the range 1 to $w-1$ using direct search method. This algorithm works well upto $n=47$ and $w=4$ for auto-correlation and cross-correlation constraints lying from 1 to 3. The second algorithm uses clique search method to find all sets of codes not only for the same length and the same weight but also for the multi-length and multi-weight one dimensional unipolar orthogonal codes. This algorithm work well upto $n=257$ and $w=5$ for auto-correlation and cross-correlation constraints lying from 1 to 2. The algorithm work well is quoted in the sense of timing required in execution of programs.

Second algorithm, proposing the codes and their all multiple sets using clique search method has reduced computational complexity. These algorithms are generating their codes in difference of positions representation (DoPR) proposed here. These codes can be converted into proper binary sequences which can be assigned to multiple users of incoherent optical cdma system.

3.2 Difference of Positions Representation (DoPR) of 1-D U(O)OC

Conventionally optical orthogonal codes are represented with their weighted positions [66],[70],[89],[92],[110],[116],[117],[128],[153] which is not a unique representation of the code because weighted positions always change with circular shift of the code. One-



dimensional uni-polar orthogonal codes are assumed to be the same with every circular shift of the code [128] for asynchronous use of the code in the multiple access systems. The difference of positions representation (DoPR) of the code remains same even with circular shift of the code. The DoPR is taken from difference families of optical orthogonal codes discussed in [140-143], [153].

Lemma 3.2.1:

The ‘ w ’ differences of consecutive weighted positions of one-dimensional uni-polar orthogonal code remain unchanged for every circular shift of the uni-polar code [128].

Proof:

The uni-polar code X with code length ‘ n ’ and weight ‘ w ’ has ‘ w ’ weighted positions. The binary code X can be put on the periphery of the circle in serial order so that last and first bits of the code are adjacent. Now on every circular shift of the binary code around the circle, the difference of second and first weighted position remains the same. Similarly for every circular shift of the code, the difference between $(j+1)^{\text{th}}$ and j^{th} weighted positions also remains the same. Finally, it can be observed that all the ‘ w ’ differences of consecutive weighted positions of the code remains unchanged on every circular shift of the code. Here $(j) \leq w$ and difference is calculated under modulo n arithmetic. \square

The lemma 3.2.1 gives the idea for unique representation of the code having ‘ w ’ differences of consecutive weighted positions of the code. These ‘ w ’ differences of consecutive weighted positions of the code is termed as difference of positions representation (DoPR) of the code. There are ‘ w ’ or less than ‘ w ’ circular shifted DoPR of the code. One of these circular shifted DoPR can be standardized to represent the code uniquely.

Example 3.2.1(a):

Let us take the code $X = [0 \ 1 \ 0 \ 1 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0 \ 1 \ 0 \ 0]$ with its WPR, $X_p = (1,3,6,10)$. The differences of consecutive weighted positions of the code are (2,3,4,4) under modulo $n = 13$ arithmetic. For every circular shifted version of code X , $(X_1, X_2, \dots, X_{12})$, the differences of consecutive weighted positions of these shifted version remain un-changed and these will be (2,3,4,4) or (3,4,4,2) or (4,4,2,3) or (4,2,3,4). The DoPR of the code X is (2,3,4,4) and the circular shifted DoPR of the code are (3,4,4,2), (4,4,2,3), (4,2,3,4).

Lemma 3.2.2:

The difference of any two weighted positions of the uni-polar code always lies from one to $(n-1)$.

Theorem 3.2.3: [153]

The sum of all the ‘ w ’ differences of consecutive weighted positions or the elements of DoPR of the uni-polar code is always equal to code length ‘ n ’.

Proof:

For the WPR of uni-polar code X , $X_p = (x_{p1}, x_{p2}, \dots, x_{pw})$.

First difference d_{x1} of positions $(x_{p1}, x_{p2}) = (x_{p2} - x_{p1})$

Second difference of positions (x_{p3}, x_{p2}) , $d_{x2} = (x_{p3} - x_{p2})$



...

$(w-1)^{\text{th}}$ difference of positions $(x_{p(w-1)}, x_{pw})$, $d_{x(w-1)} = (x_{pw} - x_{p(w-1)})$

$(w)^{\text{th}}$ difference of positions (x_{pw}, x_{p1}) , $d_{xw} = (n + x_{p1} - x_{pw})$

the $(w)^{\text{th}}$ difference d_{xw} is calculated under modulo ' n ' arithmetic because $(x_{p1} < x_{pw})$.

The sum of all ' w ' differences $= (d_{x1} + d_{x2} + \dots + d_{xw})$

$$= ((x_{pi} - x_{p1}) + (x_{pj} - x_{pi}) + \dots + (x_{pw} - x_{p(w-1)}) + (n + x_{p1} - x_{pw})) = n$$

Hence proved

□

3.2.4 Formation of Standard DoPR of the Code:

The one-dimensional uni-polar orthogonal code has a proper representation as DoPR containing ' w ' differences of consecutive positions (DoPs). The uni-polar code can be represented by any one of the ' w ' circular shifted DoPR. One of these ' w ' circular shifted DoPR can be fixed as standard DoPR following the procedure given below.

Step 1. Out of the ' w ' circular shifted DoPR, the DoPR with last element greater than other $(w-1)$ DoPs, is selected as standard DoPR of the code.

Example 3.2.4(a):

Let the uni-polar code with code length $n=31$, weight $w=5$, be $(2,5,13,4,7)$ in DoPR. The circular shifted DoPRs are $(5,13,4,7,2)$, $(13,4,7,2,5)$, $(4,7,2,5,13)$, and $(7,2,5,13,4)$. The standard DoPR of the code is $(4,7,2,5,13)$, which has highest value as last element.

Step 2. If after the step '1', the code has more than one DoPR with highest last element but equal to some DoPs of that DoPR, the DoPR with smallest value of first DoP element, is selected as standard DoPR of the code.

Example 3.2.4(b):

Let the uni-polar code with code length $n=31$, weight $w=5$, be $(6,6,7,5,7)$ in DoPR. The other circular shifted DoPRs of the code are $(6,7,5,7,6)$, $(7,5,7,6,6)$, $(5,7,6,6,7)$, $(7,6,6,7,5)$. The DoPRs selected after step 1 for standard DoPR are $(6,6,7,5,7)$ and $(5,7,6,6,7)$. The standard DoPR of the code is $(5,7,6,6,7)$ with smaller first element.

Step 3. If in the step '2' we get more than one DoPR with highest last and smallest first DoPs, the DoPR with smaller value of second DoP, is selected as standard DoPR.

Example 3.2.4(c):

Let the uni-polar code with code length $n=31$, weight $w=5$, is $(6,5,7,6,7)$ in DoPR. The other circular shifted DoPRs of the code are given as follows $(5,7,6,7,6)$, $(7,6,7,6,5)$, $(6,7,6,5,7)$, $(7,6,5,7,6)$. The DoPRs selected from step 1 for standard DoPR are $(6,5,7,6,7)$ and $(6,7,6,5,7)$. The step 2 could not standardize the code from two DoPR $(6,5,7,6,7)$ and $(6,7,6,5,7)$ of the code because first element of both DoPR is same and equal to 6. The step third results in the standard DoPR of the code as $(6,5,7,6,7)$ with smaller second DoP element out of both circular shifted DoPRs.



Step 4. The process may continue till unique and standard DoPR of the code is found, by comparing third, fourth and so on elements in the same fashion.

Lemma 3.2.5:

In the standard DoPR of the unipolar code of length ‘ n ’ and weight ‘ w ’, the range of first $\lfloor \frac{w-1}{2} \rfloor$ DoP elements lies from 1 to $\lfloor \frac{n-w+1}{2} \rfloor$ while the range of next $\lceil \frac{w-1}{2} \rceil$ DoP elements lies from 1 to $\lfloor \frac{n-w+2}{2} \rfloor$.

Proof:

Let the standard DoPR of the uni-polar code is $(d_{x1}, d_{x2}, \dots, d_{xw})$. The minimum values of all the $(d_{x1}, d_{x2}, \dots, d_{x(w-1)})$ is equal to 1 as per lemma 3.2.2. The first DoP element (d_{x1}) takes its maximum value when $(d_{x2} = d_{x3} = \dots = d_{x(w-1)}) = 1$ and $(d_{xw} > d_{x1})$ or $(d_{xw} = d_{x1} + 1)$ for standard DoPR. As per Theorem 3.2.3,

$$(d_{x1} + d_{x2} + \dots + d_{x(w-1)} + d_{xw}) = n$$

$$(d_{x1} + 1 + \dots + 1 + (d_{x1} + 1)) = n$$

$$(d_{x1} + d_{x1}) = n - (w - 1)$$

$$d_{x1} = \lfloor (n - w + 1) / 2 \rfloor.$$

Similarly (d_{x2}) or one of first $\lfloor (w-1)/2 \rfloor$ DoP elements $(d_{xi}), (1 \leq i \leq \lfloor (w-1)/2 \rfloor)$, takes its maximum value when other DoP elements except (d_{xw}) equal to one and $(d_{xw} > d_{xi})$ or $(d_{xw} = d_{xi} + 1)$ so that $d_{xi} = \lfloor (n - w + 1) / 2 \rfloor$ for standard DoPR.

One of the next remaining $\lceil (w-1)/2 \rceil$ DoP elements except last DoP element $(d_{xj}), (\lfloor (w-1)/2 \rfloor < j \leq (w-1))$, takes maximum value when other DoP elements except (d_{xw}) equal to one and $(d_{xw} \geq d_{xj})$ or $(d_{xw} = d_{xj})$ for standard DoPR. As per Theorem 3.2.3,

$$(d_{x1} + d_{x2} + \dots + d_{x(w-1)} + d_{xw}) = n$$

$$(d_{xj} + (w - 2) + d_{xj}) = n$$

$$2d_{xj} = n - (w - 2)$$

$$d_{xj} = \lfloor (n - w + 2) / 2 \rfloor.$$

If one of the first $\lfloor (w-1)/2 \rfloor$ DoP elements is equal to the last DoP element and no element of the second half $\lceil (w-1)/2 \rceil$ DoP elements is equal to the last DoP element, the code can be standardized by taking one of its circular shifted versions such that first $\lfloor (w-1)/2 \rfloor$ DoP elements is equal to the last DoP element. \square

Lemma 3.2.6:

In the standard DoPR of the uni-polar code of length ‘ n ’ and weight ‘ w ’, the last DoP element is in the range $\lceil n/w \rceil$ to $(n - w + 1)$.



Proof:

Suppose the standard DoPR of the uni-polar code is $(d_{x1}, d_{x2}, \dots, d_{xw})$. The last DoP element (d_{xw}) takes its maximum value when all other DoP elements are minimum or equal to one. Then maximum of (d_{xw}) is equal to $(n-w+1)$ as per theorem 3.2.3. The (d_{xw}) takes its minimum value when all other DoP elements are such that their DoP values are just less than or equal to last DoP element. Mathematically some of other DoP elements are equal to $\lfloor n/w \rfloor$, some are $\lceil n/w \rceil$. The minimum value of last DoP element (d_{xw}) will be $\lceil n/w \rceil$ so that it is greater than other DoP values. \square

The maximum non-zero shift auto-correlation and cross-correlation values of the codes can be calculated using the DoPR or standard DoPR. This calculation is easier than the conventional calculation of auto and cross-correlation values of the codes as given in definitions 2.3.1, 2.3.2, 2.3.3, 2.3.6, 2.3.7 & 2.3.8. For the calculation of correlation values, the DoPR is converted into extended DoP matrix of the code. The extended DoP matrix $(w \times (w-1))$ of the code contains not only differences of consecutive weighted positions but also the differences of any two weighted positions of the code.

3.2.7 Extended DoP (EDoP) Matrix of the Uni-polar Code:

- There are 'w' rows and (w-1) columns in extended DoP matrix of the code.
- The first row of extended DoP matrix contains differences of first with all other weighted positions of the code.
- The w^{th} row of extended DoP matrix contains the differences of w^{th} weighted position with all other weighted positions of the code in cyclic order.

In j^{th} row, the difference of i^{th} element with $(i+1)^{\text{st}}$ element can be placed in any column and remaining elements are placed in cyclic order. This mean for same code, we can have $(w-1)^w$ EDoP matrices. One of which may be given as follows.

Let us take the code X with DoPR $(d_{x1}, d_{x2}, \dots, d_{xw})$ with weight 'w' and code length $n = d_{x1} + d_{x2} + \dots + d_{xw}$, the EDoP matrix is formed as follows.

$$\text{EDoP} \begin{bmatrix} e_{x01} & e_{x02} & \dots & e_{x0(w-2)} & e_{x0(w-1)} \\ e_{x11} & e_{x12} & \dots & e_{x1(w-2)} & e_{x1(w-1)} \\ \dots & \dots & \dots & \dots & \dots \\ e_{x(w-2)1} & e_{x(w-2)2} & \dots & e_{x(w-2)(w-2)} & e_{x(w-2)(w-1)} \\ e_{x(w-1)1} & e_{x(w-1)2} & \dots & e_{x(w-1)(w-2)} & e_{x(w-1)(w-1)} \end{bmatrix}$$



$$e_{x01} = d_{x1}; e_{x11} = d_{x2}; \dots; e_{x(w-2)1} = d_{x(w-1)}; e_{x(w-1)1} = d_{xw}.$$

$$e_{x02} = d_{x1} + d_{x2}; e_{x12} = d_{x2} + d_{x3};$$

...

$$e_{x(w-2)2} = d_{x(w-1)} + d_{xw}; e_{x(w-1)2} = d_{xw} + d_{x1};$$

...

$$e_{x0(w-2)} = d_{x1} + d_{x2} + \dots + d_{x(w-2)}; e_{x1(w-2)} = d_{x2} + d_{x3} + \dots + d_{x(w-1)};$$

...

$$e_{x(w-2)(w-2)} = d_{x(w-1)} + d_{xw} + d_{x1} + d_{x2} + \dots + d_{x(w-4)};$$

$$e_{x(w-1)(w-2)} = d_{xw} + d_{x1} + d_{x2} + \dots + d_{x(w-3)};$$

$$e_{x0(w-1)} = d_{x1} + d_{x2} + \dots + d_{x(w-1)}; e_{x1(w-1)} = d_{x2} + d_{x3} + \dots + d_{xw};$$

...

$$e_{x(w-2)(w-1)} = d_{x(w-1)} + d_{xw} + d_{x1} + d_{x2} + \dots + d_{x(w-3)};$$

$$e_{x(w-1)(w-1)} = d_{xw} + d_{x1} + d_{x2} + \dots + d_{x(w-2)}.$$

Example 3.2.7(a):

Let the DoPR of the code with weight 'w' equal to 5 is (a,b,c,d,e) and code length 'n'=a+b+c+d+e (as per theorem 3.2.3). The extended DoP matrix (5x4) is given as

$$\begin{bmatrix} a & a+b & a+b+c & a+b+c+d \\ b & b+c & b+c+d & b+c+d+e \\ c & c+d & c+d+e & c+d+e+a \\ d & d+e & d+e+a & d+e+a+b \\ e & e+a & e+a+b & e+a+b+c \end{bmatrix}.$$

Lemma 3.2.8:

If 'a' is a DoP element of extended DoP matrix of the code, then the DoP element 'n-a' also exist in the same extended DoP matrix of the code.

Proof:

if 'a' is a difference of any two weighted positions (x_{pi}, x_{pj}) of the code such that $(i, j) \in (0 : n-1)$. i.e. $a = (x_{pj} - x_{pi})$, while the difference between (x_{pj}, x_{pi}) in circular order is $(x_{pi} - x_{pj}) = (n-a)$ in modulo 'n' arithmetic. It means that two DoP elements 'a' and 'n-a' represents the difference of two same weighted positions.

As well as in example 3.2.7(a), n-a=b+c+d+e, which is one element of extended DoP matrix of the code (a,b,c,d,e) in DoPR. □



Lemma 3.2.9:

If first $(w-1)$ consecutive differences of weighted positions or DoP element $(d_{x1}, d_{x2}, \dots, d_{x(w-1)})$ of DoPR $(d_{x1}, d_{x2}, \dots, d_{x(w-1)}, d_{xw})$ of the code are known, the extended DoP matrix is given as follows.

$$\text{EDoP} \begin{bmatrix} e_{x01} & e_{x02} & \cdots & e_{x0(w-2)} & e_{x0(w-1)} \\ e_{x11} & e_{x12} & \cdots & e_{x1(w-2)} & e_{x1(w-1)} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ e_{x(w-2)1} & e_{x(w-2)2} & \cdots & e_{x(w-2)(w-2)} & e_{x(w-2)(w-1)} \\ e_{x(w-1)1} & e_{x(w-1)2} & \cdots & e_{x(w-1)(w-2)} & e_{x(w-1)(w-1)} \end{bmatrix}$$

With

$$e_{x01} = d_{x1}; \quad e_{x11} = d_{x2};$$

...

$$e_{x(w-2)1} = d_{x(w-1)};$$

$$e_{x(w-1)1} = d_{xw} = (n - (d_{x1} + d_{x2} + \dots + d_{x(w-1)}));$$

$$e_{x02} = d_{x1} + d_{x2}; \quad e_{x12} = d_{x2} + d_{x3};$$

...

$$e_{x(w-2)2} = d_{x(w-1)} + d_{xw} = (n - (d_{x1} + d_{x2} + \dots + d_{x(w-2)}));$$

$$e_{x(w-1)2} = d_{xw} + d_{x1} = (n - (d_{x2} + d_{x3} + \dots + d_{x(w-1)}));$$

...

$$e_{x0(w-2)} = d_{x1} + d_{x2} + \dots + d_{x(w-2)};$$

$$e_{x1(w-2)} = d_{x2} + d_{x3} + \dots + d_{x(w-1)};$$

...

$$e_{x(w-2)(w-2)} = d_{x(w-1)} + d_{xw} + d_{x1} + d_{x2} + \dots + d_{x(w-4)} = (n - (d_{x(w-3)} + d_{x(w-2)}));$$

$$e_{x(w-1)(w-2)} = d_{xw} + d_{x1} + d_{x2} + \dots + d_{x(w-3)} = (n - (d_{x(w-2)} + d_{x(w-1)}));$$

$$e_{x0(w-1)} = d_{x1} + d_{x2} + \dots + d_{x(w-1)}; e_{x1(w-1)} = d_{x2} + d_{x3} + \dots + d_{xw} = (n - d_{x1}); \dots;$$

$$e_{x(w-2)(w-1)} = d_{x(w-1)} + d_{xw} + d_{x1} + d_{x2} + \dots + d_{x(w-3)} = (n - d_{x(w-2)});$$

$$e_{x(w-1)(w-1)} = d_{xw} + d_{x1} + d_{x2} + \dots + d_{x(w-2)} = (n - d_{x(w-1)}).$$



Example 3.2.9(a):

Let the DoPR of the code with weight 'w' equal to 5 is (a,b,c,d,e) and code length 'n'=a+b+c+d+e. The extended DoP matrix (5x4) is given as

$$\begin{bmatrix} a & a+b & a+b+c & a+b+c+d \\ b & b+c & b+c+d & n-a \\ c & c+d & n-(a+b) & n-b \\ d & n-(a+b+c) & n-(b+c) & n-c \\ n-(a+b+c+d) & n-(b+c+d) & n-(c+d) & n-d \end{bmatrix}$$

Lemma 3.2.10:

If first u ($u < w$) consecutive differences of weighted positions or DoP element ($d_{x1}, d_{x2}, \dots, d_{xu}$) of DoPR ($d_{x1}, d_{x2}, \dots, d_{x(w-1)}, d_{xw}$) of the code are known, the extended DoP matrix for the incomplete code with u DoP elements is given as follows.

$$\text{EDoP} \begin{bmatrix} e_{x01} & e_{x02} & \dots & e_{x0(u-1)} & e_{x0u} \\ e_{x11} & e_{x12} & \dots & e_{x1(u-1)} & e_{x1u} \\ \dots & \dots & \dots & \dots & \dots \\ e_{x(u-1)1} & e_{x(u-1)2} & \dots & e_{x(u-1)(u-1)} & e_{x(u-1)u} \\ e_{x(u)1} & e_{x(u)2} & \dots & e_{x(u)(u-1)} & e_{x(u)u} \end{bmatrix}$$

$$\text{With} \begin{bmatrix} e_{x01} = d_{x1}; \\ e_{x11} = n - d_{x1} \end{bmatrix}$$

$$\begin{bmatrix} e_{x02} = d_{x1} + d_{x2}; e_{x12} = d_{x2} \\ e_{x21} = n - (d_{x1} + d_{x2}); e_{x22} = n - d_{x2} \end{bmatrix}$$

$$\begin{bmatrix} e_{x03} = (d_{x1} + d_{x2} + d_{x3}); e_{x13} = (d_{x2} + d_{x3}); e_{x23} = d_{x3}; \\ e_{x31} = n - (d_{x1} + d_{x2} + d_{x3}); e_{x32} = n - (d_{x2} + d_{x3}); e_{x33} = n - d_{x3} \end{bmatrix}; \dots;$$

$$\begin{bmatrix} e_{x0(u)} = (d_{x1} + d_{x2} + \dots + d_{xu}); e_{x1(u)} = (d_{x2} + d_{x3} + \dots + d_{xu}); \\ e_{x2(u)} = (d_{x3} + d_{x4} + \dots + d_{xu}); \dots; e_{x(u-1)(u)} = (d_{xu}); \\ e_{xu1} = n - (d_{x1} + d_{x2} + \dots + d_{xu}); e_{xu2} = n - (d_{x2} + d_{x3} + \dots + d_{xu}) \\ \dots; e_{x(u)(u-1)} = n - (d_{x(u-1)} + d_{xu}); e_{x(u)u} = n - (d_{xu}); \end{bmatrix}$$

Proof:

It is obvious that there is no change in EDoP matrix if EDoP element in the same row moves to another position. The EDoP matrix of Lemma 3.9 is same as EDoP matrix of lemma 3.10 with cyclic shift within rows. It can be verified by examples 3.9(a) and 3.10(a) with i^{th} row being cyclically shifted left (i-1) times. The advantage of this kind of EDoP representation is that if last k entries of DoPR are deleted, EDoP can be determined by deleting lower k rows and rightmost k column in EDoP representation of complete code. \square



Example 3.2.10(a):

Let the DoPR of the code with weight 'w' equal to 5 is (a,b,c,d,e) and code length $n = (a+b+c+d+e)$. The extended DoP matrix for 1, 2, 3, and 4 consecutive DoP elements of incomplete and complete code is given as following matrices respectively.

$$\begin{bmatrix} a \\ n-a \end{bmatrix}, \begin{bmatrix} a & a+b \\ n-a & b \\ n-(a+b) & n-b \end{bmatrix}, \begin{bmatrix} a & a+b & a+b+c \\ n-a & b & b+c \\ n-(a+b) & n-b & c \\ n-(a+b+c) & n-(b+c) & n-c \end{bmatrix},$$

$$\begin{bmatrix} a & a+b & a+b+c & a+b+c+d \\ n-a & b & b+c & b+c+d \\ n-(a+b) & n-b & c & c+d \\ n-(a+b+c) & n-(b+c) & n-c & d \\ n-(a+b+c+d) & n-(b+c+d) & n-(c+d) & n-d \end{bmatrix}.$$

3.3 THE CALCULATION OF CORRELATION CONSTRAINTS**3.3.1 Auto-Correlation Constraint:**

In the conventional method for calculation of maximum non-zero shift auto-correlation as given in definition 2.3.1, the weighted bits' positions of code X are compared with circular shifted versions of code X. There will be $n(n-1)$ comparisons of binary digits in the calculation of maximum non-zero shift auto-correlation of uni-polar code as given in definition 2.3.1. The 'n' bits of code X are compared with 'n' bits of each of (n-1) circular shifted versions of code X. These comparisons of weighted bits positions can be reduced as described below.

Lemma 3.3.1.1:

In calculation of the maximum non-zero shift auto-correlation using weighted positions representation (WPR) of the code, there are $(n-1)w^2$ comparisons of weighted positions (Definition 2.3.3).

Proof:

In conventional method for the calculation of the maximum non-zero shift auto-correlation of the uni-polar code, the 'w' weighted positions of X_p are compared with 'w' weighted positions of each of the (n-1) circular shifted versions (X_{p+a}). Thus, there are $(n-1)w^2$ comparisons of weighted positions in the calculation of the maximum non-zero shift auto-correlation of the code X. □

Lemma 3.3.1.2:

For the uni-polar code of length 'n' and weight 'w', the total definite cases of overlapping of weighted bits of uni-polar code with its circular shifted versions in the calculation



of maximum non-zero shift auto-correlation (Definition 2.3.1) are $\frac{w(w-1)}{2}$.

Proof: In the calculation of maximum non-zero shift auto-correlation, first weighted bit of the uni-polar code overlap with next $(w-1)$ other weighted bits by circular shifting. The second weighted bit overlap with next $(w-2)$ weighted bits by circular shifting. Similarly the third and so on up to $(w-1)^{\text{th}}$ weighted bit overlap with next $(w-3)$ and so on up to last weighted bit by circular shifting. There are total $(w-1)$ plus $(w-2)$ plus $(w-3)$ plus and so on up to plus one overlapping occurred in the pairs of codes with its maximum $(n-1)$ circular shifted versions. These total overlapping of weighted bits are $w(w-1)/2$. \square

Lemma 3.3.1.3:

The uni-polar code with code length ' n ' and code weight ' w ' has ' w ' circular shifted versions with first bit as weighted bit of the code.

Example 3.3.1.3(a):

Let us take the code $X = [0 \ 1 \ 0 \ 1 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0 \ 1 \ 0 \ 0]$ with weighted positions representation $X_p = (1,3,6,10)$. The $w=4$ circular shifted versions of the code with first bit as weighted bit are given as follows

$$\begin{aligned} X_1 &= [1 \ 0 \ 1 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0], (X_p+12) = (0,2,5,9), \\ X_3 &= [1 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0 \ 1 \ 0], (X_p+10) = (0,3,7,11), \\ X_6 &= [1 \ 0 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0 \ 1 \ 0 \ 1 \ 0 \ 0], (X_p+7) = (0,4,8,10), \\ X_{10} &= [1 \ 0 \ 0 \ 0 \ 1 \ 0 \ 1 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0], (X_p+3) = (0,4,6,9). \square \end{aligned}$$

Lemma 3.3.1.4:

There are ${}^w C_2 = \frac{w(w-1)}{2}$ pairs of codes formed out of the ' w ' circular shifted versions of the code with first bit as weighted bit. \square

Lemma 3.3.1.5:

The definite overlapping of weighted bits in the all pairs of codes having first bit as weighted bit are $\frac{w(w-1)}{2}$.

Proof:

As per lemma 3.3.1.4, there are $\frac{w(w-1)}{2}$ pairs of codes having first bit as weighted bit. Each pair has one definite overlapping at first position. Then there are $\frac{w(w-1)}{2}$ definite overlapping of weighted bits. \square

Theorem 3.3.1.6:

The overlapping of weighted bits of uni-polar code with its every circular shifted version equal to overlapping of weighted bits in all the pairs of the ' w ' circular shifted versions with first bit as weighted bit of the code.



Proof:

As per lemma 3.3.1.2, the definite overlapping of weighted bits of uni-polar code with its every circular shifted version is $\frac{w(w-1)}{2}$. As per lemma 3.3.1.5, the same number of definite overlapping are found in all the pairs of the 'w' circular shifted versions with first bit as weighted bit of the code. Hence all definite weighted overlapping are covered in both cases. \square

Theorem 3.3.1.7:

The weighted positions of the 'w' circular shifted versions of the code with first bit as weighted bit are given by the rows of EDoP matrix along-with extra first column having zero elements.

Proof:

Let us take the code X with DoPR (a, b, c, d, e) with weight $w=5$, and code length $n=(a+b+c+d+e)$. The weighted positions of the code with first bit as weighted bit are $(0, a, a+b, a+b+c, a+b+c+d)$. The circular shifted versions of this code with first bit as weighted bit are $(0, b, b+c, b+c+d, b+c+d+e)$, $(0, c, c+d, c+d+e, c+d+e+a)$, $(0, d, d+a, d+a+b, d+a+b+c)$ and $(0, e, e+a, e+a+b, e+a+b+c)$. These circular shifted versions of the code with first bit as weighted bit are same as row element of the following EDoP matrix (example 3.2.7(a)) along-with extra first column having zero elements.

$$\begin{bmatrix} 0 & a & a+b & a+b+c & a+b+c+d \\ 0 & b & b+c & b+c+d & b+c+d+e \\ 0 & c & c+d & c+d+e & c+d+e+a \\ 0 & d & d+e & d+e+a & d+e+a+b \\ 0 & e & e+a & e+a+b & e+a+b+c \end{bmatrix},$$

Similarly for any weight $w \geq 2$, the theorem can be verified easily. \square

Theorem 3.3.1.8:

The maximum non-zero shift auto-correlation of the uni-polar code is equal to maximum number of overlapping bits among the pairs of 'w' circular shifted versions with first bit as weighted bit of the code.

OR

The maximum non-zero shift auto-correlation of the uni-polar code is equal to the maximum number of common DoP elements between two rows of EDoP matrix having zero elements in first column. $\lambda_{ax} \geq \sum_{j=0}^{w-1} \sum_{l=0}^{w-1} e_{xij} e_{xkl}$ for $i = (0: w-1), k = (i+1: w-1)$

OR

The maximum non-zero shift auto-correlation of the uni-polar code is equal to one plus maximum number of common DoP elements between two rows of EDoP matrix of the code.

$$\lambda_{ax} \geq 1 + \sum_{j=1}^{w-1} \sum_{l=1}^{w-1} e_{xij} e_{xkl} \text{ for } i = (0: w-1), k = (i+1: w-1) \text{ where } e_{xij} e_{xkl} = \begin{cases} 1 & \text{if } e_{xij} = e_{xkl} \\ 0 & \text{if } e_{xij} \neq e_{xkl} \end{cases}$$

e_{xij} & e_{xkl} are DoP elements of two rows of EDoP matrix.



Proof:

Let us take the code X with DoPR $(d_{x1}, d_{x2}, \dots, d_{xw})$ with weight 'w' and code length $n = d_{x1} + d_{x2} + \dots + d_{xw}$, the EDoP matrix with zero elements in the first column is formed as follows.

$$\text{EDoP} \begin{bmatrix} e_{x00} & e_{x01} & e_{x02} & \dots & e_{x0(w-1)} \\ e_{x10} & e_{x11} & e_{x12} & \dots & e_{x1(w-1)} \\ e_{x20} & e_{x21} & e_{x22} & \dots & e_{x2(w-1)} \\ \dots & \dots & \dots & \dots & \dots \\ e_{x(w-1)0} & e_{x(w-1)1} & e_{x(w-1)2} & \dots & e_{x(w-1)(w-1)} \end{bmatrix}$$

$$e_{x00} = e_{x10} = e_{x20} = \dots = e_{x(w-1)0} = 0.$$

$$e_{x01} = d_{x1}; e_{x11} = d_{x2}; \dots; e_{x(w-1)1} = d_{xw}.$$

with $e_{x02} = d_{x1} + d_{x2}; e_{x12} = d_{x2} + d_{x3}; \dots; e_{x(w-1)2} = d_{xw} + d_{x1}$

...

$$e_{x0(w-1)} = d_{x1} + d_{x2} + \dots + d_{xw-1}; e_{x1(w-1)} = d_{x2} + d_{x3} + \dots + d_{xw}; \dots;$$

$$e_{x(w-1)(w-1)} = d_{xw} + d_{x1} + d_{x2} + \dots + d_{x(w-2)}.$$

As per definition 2.3.1 and theorem 3.3.1.6, the maximum non-zero shift auto-correlation λ_{ax} of the uni-polar code is equal to maximum number of overlapping of weighted bits among the pairs of circular shifted versions with first bit as weighted bit of the code. However As per theorem 3.3.1.7 and definition 2.3.3, the maximum non-zero shift auto-correlation of the code is equal to the maximum number of common DoP elements between two

rows of EdoP matrix along-with first column with zero elements. if $e_{xij}e_{xkl} = \begin{cases} 1 & \text{if } e_{xij} = e_{xkl} \\ 0 & \text{if } e_{xij} \neq e_{xkl} \end{cases}$

which represents common elements between two rows of EdoP matrix with first column having

zero elements. $\lambda_{ax} \geq \sum_{j=0}^{w-1} \sum_{l=0}^{w-1} e_{xij}e_{xkl}$ for $i = (0:w-1), k = (i+1:w-1)$ Or the maximum non-zero

shift auto-correlation of the uni-polar code is equal to one plus maximum common DoP elements between two rows of EDoP matrix of the code. As any two rows of EdoP matrix with first column having zero elements always has at least one common element which is zero.

$$\lambda_{ax} \geq 1 + \sum_{j=1}^{w-1} \sum_{l=1}^{w-1} e_{xij}e_{xkl} \text{ for } i = (0:w-1), k = (i+1:w-1) \quad \square$$

Lemma 3.3.1.9:

There will be $\frac{(w-1)w^3}{2}$ comparisons of DoP elements in the calculation of

maximum non-zero shift auto-correlation using extended DoP matrix with first column having zero elements.

Proof: There are $\frac{w(w-1)}{2}$ pair of rows of extended DoP matrix which are compared in the



calculation of maximum non-zero shift auto-correlation of the code. In each pair of rows, there are w^2 comparisons of DoP elements. Thus there are total $\frac{(w-1)w^3}{2}$ comparisons of DoP elements of EDoP matrix take place in the calculation of maximum non-zero shift auto-correlation of the code. \square

Lemma 3.3.1.10:

There are $\frac{w(w-1)^3}{2}$ comparisons of DoP elements in the calculation of maximum non-zero shift auto-correlation using extended DoP matrix of the code.

Proof: There are $\frac{w(w-1)}{2}$ pair of rows of extended DoP matrix which are compared in the calculation of maximum non-zero shift auto-correlation of the code. In each pair of rows, there are $(w-1)^2$ comparisons of DoP elements. Hence there are $\frac{w(w-1)^3}{2}$ total comparisons of DoP elements of EDoP matrix in the calculation of maximum non-zero shift auto-correlation of the code. \square

Lemma 3.3.1.11: [147]

If there are no common DoP elements in the pair of rows of EDoP matrix of code, the maximum non-zero shift auto-correlation of the code is always equals to one.

3.3.2 Cross-Correlation Constraint:

In the conventional method for calculation of cross-correlation for the pair of uni-polar codes as given in definition 2.3.6, the weighted bits' positions of code X are compared with code Y and circular shifted versions of code Y. Or the weighted bits' positions of code Y are compared with code X and circular shifted versions of code X. There are n^2 comparisons of binary digits in the calculation of cross-correlation of uni-polar code in conventional method (definition 2.3.6). These comparisons of weighted bits positions can be further reduced as described below.

Lemma 3.3.2.1:

In the calculation of cross-correlation using weighted positions representation (WPR) of the pair of codes, there are (nw^2) comparisons of weighted positions (in definition 2.3.8).

Proof:

In the calculation of cross-correlation of the pair of uni-polar codes (definition 2.3.8), the ' w ' weighted positions (WP) of X_P are compared with ' w ' weighted positions of Y_P and each of the $(n-1)$ circular shifted versions (Y_{P+a}). There are (nw^2) total comparisons of weighted position in the calculation of cross-correlation of the pair of codes. \square



Lemma 3.3.2.2:

For the uni-polar codes of length ' n ' and weight ' w ', the definite cases of overlapping of weighted bits of uni-polar code X with code Y and the $(n-1)$ circular shifted versions of code Y are w^2 .

Proof:

In the calculation of cross-correlation (definition 2.3.6), first weighted bit of code X overlap with w weighted bits of code Y in ' w ' shifts. The second weighted bit of code X overlap with ' w ' weighted bit of code Y in ' w ' shifts. Similarly the third and so on upto w^{th} weighted bit of code X overlap with ' w ' weighted positions of code Y in ' w ' shifts. Thus there are (w^2) total overlapping of weighted bits occurred in the pairs of code X with code Y and the maximum $(n-1)$ circular shifted versions of code Y in the calculation of cross-correlation. \square

Lemma 3.3.2.3:

There are total w^2 pairs of code X and code Y formed out of the ' w ' circular shifted versions of both the codes with first bit as weighted bit. \square

Lemma 3.3.2.4:

The definite overlapping of weighted bits in all the pairs of circular shifted versions of codes X and Y having first bit as weighted bit are w^2 .

Proof: As per lemma 3.3.2.3, there are w^2 pairs of codes having first bit as weighted bit. Each pair has one definite overlapping at first position. Subsequently, there is w^2 definite overlapping of weighted bits. \square

Theorem 3.3.2.5:

The overlapping of weighted bits of uni-polar code X with uni-polar code Y and every circular shifted version of code Y equals to the overlapping of weighted bits in all the pairs of code X and code Y formed out of the ' w ' circular shifted versions of both the codes having first bit as weighted bit.

Proof:

As per lemma 3.3.2.2, the definite overlapping of weighted bits of uni-polar code X and code Y along-with every circular shifted version of code Y is w^2 . As well as per lemma 3.3.2.4, the same number of definite overlapping are covered in all the pairs of circular shifted versions of code X and code Y having first bit as weighted bit. Hence all definite weighted overlapping are covered in both cases. \square

Theorem 3.3.2.6:

The cross-correlation of the uni-polar codes X and Y is equal to maximum overlapping among the pairs of code X and code Y out of the ' w ' circular shifted versions with first bit as weighted bit of both the codes.

OR

The cross-correlation of the uni-polar codes X and Y is equal to maximum common DoP elements between any two rows of EdoP matrices along-with first column with zero elements of



code X and code Y respectively. $\lambda_{cxy} \geq \sum_{j=0}^{w-1} \sum_{l=0}^{w-1} e_{xij} e_{ykl}$, for $i = (0:w-1), k = (0:w-1)$

OR

The cross-correlation of the uni-polar codes X and Y is equal to one plus maximum common DoP elements between any two rows of EDoP matrices of code X and code Y respectively.

$$\lambda_{cxy} \geq 1 + \sum_{j=1}^{w-1} \sum_{l=1}^{w-1} e_{xij} e_{ykl}, \text{ for } i = (0:w-1), k = (0:w-1) \text{ where } e_{xij} e_{ykl} = \begin{cases} 1 & \text{if } e_{xij} = e_{ykl} \\ 0 & \text{if } e_{xij} \neq e_{ykl} \end{cases}$$

e_{xij} & e_{ykl} are DoP elements of the rows of EDoP matrices along-with extra column with zero elements of code X and code Y respectively.

Proof: Suppose the code X with DoPR $(d_{x1}, d_{x2}, \dots, d_{xw})$ and code Y with DoPR $(d_{y1}, d_{y2}, \dots, d_{yw})$ with weight 'w' and code length $n = d_{x1} + d_{x2} + \dots + d_{xw} = d_{y1} + d_{y2} + \dots + d_{yw}$, the EDoP matrix along-with first column with zero elements of code X and code Y are formed as follows

$$\text{EDoP(X)} \begin{bmatrix} e_{x00} & e_{x01} & e_{x02} & \dots & e_{x0(w-1)} \\ e_{x10} & e_{x11} & e_{x12} & \dots & e_{x1(w-1)} \\ e_{x20} & e_{x21} & e_{x22} & \dots & e_{x2(w-1)} \\ \dots & \dots & \dots & \dots & \dots \\ e_{x(w-1)0} & e_{x(w-1)1} & e_{x(w-1)2} & \dots & e_{x(w-1)(w-1)} \end{bmatrix}$$

With

$$e_{x00} = e_{x10} = e_{x20} = \dots = e_{x(w-1)0} = 0.$$

$$e_{x01} = d_{x1}; e_{x11} = d_{x2};$$

...

$$e_{x(w-1)1} = d_{xw}.$$

$$e_{x02} = d_{x1} + d_{x2}; e_{x12} = d_{x2} + d_{x3};$$

...

$$e_{x(w-1)2} = d_{xw} + d_{x1}$$

...

$$e_{x0(w-1)} = d_{x1} + d_{x2} + \dots + d_{x(w-1)};$$

$$e_{x1(w-1)} = d_{x2} + d_{x3} + \dots + d_{xw};$$

...

$$e_{x(w-1)(w-1)} = d_{xw} + d_{x1} + d_{x2} + \dots + d_{x(w-2)}.$$



$$\text{and EDoP(Y)} \begin{bmatrix} e_{y00} & e_{y01} & e_{y02} & \cdots & e_{y0(w-1)} \\ e_{y10} & e_{y11} & e_{y12} & \cdots & e_{y1(w-1)} \\ e_{y20} & e_{y21} & e_{y22} & \cdots & e_{y2(w-1)} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ e_{y(w-1)0} & e_{y(w-1)1} & e_{y(w-1)2} & \cdots & e_{y(w-1)(w-1)} \end{bmatrix}$$

$$e_{y00} = e_{y10} = e_{y20} = \dots = e_{y(w-1)0} = 0.$$

$$e_{y01} = d_{y1}; e_{y11} = d_{y2};$$

...

$$e_{y(w-1)1} = d_{yw}.$$

$$e_{y02} = d_{y1} + d_{y2}; e_{y12} = d_{y2} + d_{y3};$$

...

with

$$e_{y(w-1)2} = d_{yw} + d_{y1}$$

...

$$e_{y0(w-1)} = d_{y1} + d_{y2} + \dots + d_{y(w-1)};$$

$$e_{y1(w-1)} = d_{y2} + d_{y3} + \dots + d_{yw};$$

...

$$e_{y(w-1)(w-1)} = d_{yw} + d_{y1} + d_{y2} + \dots + d_{y(w-2)}.$$

As per definition 2.3.6 and theorem 3.3.2.5, the cross-correlation λ_{cxy} of the uni-polar codes X and Y is equal to the maximum number of overlapping among the pairs of code X and code Y of the circular shifted versions with first bit as weighted bit of both the codes. However As per definition 2.3.6 and theorem 3.3.1.7, the cross-correlation of the codes X and Y is the maximum number of common DoP elements between the rows of EDoP matrices along-with first column having zero elements for both the codes X and Y.

$$\text{if } e_{xij}e_{ykl} = \begin{cases} 1 & \text{if } e_{xij} = e_{ykl} \\ 0 & \text{if } e_{xij} \neq e_{ykl} \end{cases} .$$

$$\lambda_{cxy} \geq \sum_{j=0}^{w-1} \sum_{l=0}^{w-1} e_{xij}e_{ykl} , \text{ for } i = (0 : w-1), k = (0 : w-1)$$

where e_{xij} and e_{ykl} are the DoP elements of the rows of EDoP matrices along-with first extra column having zero elements of code X and code Y respectively..

Or the cross-correlation of the uni-polar codes X and Y is equal to one plus maximum common DoP elements between the rows of EDoP matrices of the code X and Y. Because any two rows of EdoP matrices along-with first column having zero elements for code X and Y always has at least one common element as zero.

$$\lambda_{cxy} \geq 1 + \sum_{j=1}^{w-1} \sum_{l=1}^{w-1} e_{xij}e_{ykl} , \text{ for } i = (0 : w-1), k = (0 : w-1) \square$$



Lemma 3.3.2.7:

There are w^4 comparisons of DoP elements in the calculation of cross-correlation using extended DoP matrices along-with first column having zero elements for both the codes.

Proof: As per lemma 3.3.2.3, there are w^2 pair of rows from extended DoP matrices along-with first column having zero elements for code X and Y. In each pair of rows, there are w^2 comparisons of DoP elements. Hence there are w^4 comparisons of DoP elements of both EDoP matrices in the calculation of cross-correlation for the pair of codes X and Y. \square

Lemma 3.3.2.8:

In calculation of cross-correlation using extended DoP matrices of the codes X and Y, there are $w^2(w-1)^2$ comparisons of DoP elements.

Proof: As per lemma 3.3.2.3, there are w^2 pair of rows from extended DoP matrices of code X and Y. In each pair of rows, there are $(w-1)^2$ comparisons of DoP elements. Hence there are $w^2(w-1)^2$ total comparisons of DoP elements of EDoP matrices in the calculation of cross-correlation of the codes X with Y. \square

Lemma 3.3.2.9:

If there are no common DoP elements in the pair of rows of EDoP matrices of the two codes, the cross-correlation of the pair of codes is always equals to one [23]. \square

Theorem 3.3.2.10:

The cross-correlation of the uni-polar code X with code parameters (n_1, w_1, λ_{a1}) and code Y with parameters (n_2, w_2, λ_{a2}) is equal to maximum common DoP elements between the any two rows of EdoP matrices along-with first column having zero elements of code X and code Y respectively.

$$\lambda_{cxy} \geq \sum_{j=0}^{w_1-1} \sum_{l=0}^{w_2-1} e_{xij} e_{ykl}, \text{ for } i = (0 : w_1 - 1), k = (0 : w_2 - 1)$$

OR

The cross-correlation of the uni-polar codes X with code parameters (n_1, w_1, λ_{a1}) and code Y with parameters (n_2, w_2, λ_{a2}) is equal to one plus maximum common DoP elements between any two rows of EDoP matrices of code X and code Y.

$$\lambda_{cxy} \geq 1 + \sum_{j=1}^{w_1-1} \sum_{l=1}^{w_2-1} e_{xij} e_{ykl}, \text{ for } i = (0 : w_1 - 1), k = (0 : w_2 - 1) \text{ where } e_{xij} e_{ykl} = \begin{cases} 1 & \text{if } e_{xij} = e_{ykl} \\ 0 & \text{if } e_{xij} \neq e_{ykl} \end{cases}$$

e_{xij} & e_{ykl} are DoP elements of the rows of EDoP matrices along-with first column having zero elements of code X and code Y respectively.

Proof: it is straight forward through theorem 3.3.2.6.



3.4 Design of the Maximal Sets of 1-DUOC:

The maximal sets of 1-DUOC for fixed code parameters $(n, w, \lambda_a, \lambda_c)$ can be designed using anyone of the two proposed algorithms.

3.4.1 Algorithm One to design the maximum sets of 1-DUOC:

The algorithm one can generate all possible multiple sets of one dimensional unipolar orthogonal codes for given code length ' n ', code weight ' w ' and correlation constraints lying from 1 to $w-1$, such that w and n are co-prime (no common factors) and $w^2 < n$. The codes are generated in difference of positions representation (DoPR). The steps of algorithm one are as follows.

Step-1: Input code length ' n ', code weight ' w ', auto-correlation constraint ' λ_a ' and cross-correlation constraint ' λ_c ' for the code sets to be generated.

Step-2: Initialize w variables $(a_1, a_2, \dots, a_{w-1})$ equal to one and $a_w = (n - (a_1 + a_2 + \dots + a_{w-1}))$.

Step-3: Generate all the codes of set (n, w) in standardized DoPR in sequence starting from $(1, 1, \dots, 1, n - w + 1)$ to (a_1, a_2, \dots, a_w) with enumeration.

$$(i) \ a_w \geq (a_1, a_2, \dots, a_{w-1}) \geq 1 \quad (ii) \ \left\lceil \frac{n}{w} \right\rceil \leq a_w \leq (n - w + 1).$$

The variables $(a_1, a_2, \dots, a_{w-1}, a_w)$ in DoPR, represent the difference of weighted positions or position of bit 1's in serial and circular order in the code.

All the codes generated with condition $a_w > (a_1, a_2, \dots, a_{w-1})$ will always be in standard DoP representation. While for the condition when a_w is equal to any one or more than one of $(a_1, a_2, \dots, a_{w-1}, a_w)$ and greater than remaining DoP elements, the code has more than one representations as $a_w \geq (a_1, a_2, \dots, a_{w-1})$ out of their circular shifted versions. In this condition, that representation is chosen for which (i) a_1 is minimum, and (ii) If minimum a_1 is found in more than one DoP representations, then minimum a_2 is searched among DoPs with minimum a_1 . The DoP representation with minimum a_1 and minimum a_2 is considered as standard DoP representation. Similarly, search upto a_{w-1} to find standard DoP representation may be needed if $(a_1, a_2, \dots, a_{w-2})$ are same in more than two members of candidate codes. The upper bound of the set (n, w) of these generated unipolar orthogonal codes is equal to Johnson bound for the set of unipolar orthogonal codes with maximum correlation constraints



($\lambda_a = \lambda_c = w-1$). These generated unipolar codes in DoPR are numbered serially from Code#1 to Code #N for identification of codes. N is maximum number of codes generated.

$$N = \left\lfloor \frac{(n-1)(n-2)\dots(n-w+1)}{w(w-1)\dots 2.1} \right\rfloor$$

here $\lfloor a \rfloor$ represent integer value just less than a, and $\lceil a \rceil$ represent integer value just greater than a.

Alternative to step 3:

As per lemma 3.2.5, the first $\lfloor \frac{w-1}{2} \rfloor$ DoP elements are varied in the range 1 to $\lfloor \frac{n-w+1}{2} \rfloor$ while keeping next $\lceil \frac{w-1}{2} \rceil$ DoP elements in the range 1 to $\lfloor \frac{n-w+2}{2} \rfloor$ while keeping last DoP element in the range $\lceil n/w \rceil$ to $(n-w+1)$ as per lemma 3.2.6. It generates all the code in standard DoPR automatically.

Step 4: Calculation of auto-correlation constraints

For the generated codes in DoPR in step 3, the auto-correlation constraint of each code can be calculated through the use of proposed method for calculation of correlation constraints described in theorem 3.3.1.8 earlier.

Step 5: Calculation of cross-correlation constraints

The cross-correlation constraint for each pair of unipolar orthogonal codes generated in step-3, is calculated through the use of proposed method described earlier in theorem 3.3.2.10. The cross-correlation for each pair containing code#1 with code of code number greater than 1, secondly the code#2 with code of code number greater than 2, upto code#(N-1) with code#N.

Step 6: Formation of correlation matrix

In step 3, the number of generated codes are N . A $N \times N$ matrix can be formed in such a way that it contains correlation of code# x with code# y , for $1 \leq (x, y) \leq N$.

When $x = y$, it represent maximum auto-correlation for non zero shift or auto-correlation constraint of code# x or code# y , which form diagonal elements of $N \times N$ correlation matrix. For $x \neq y$, cross correlation constraint of code# x with code# y is found as a non-diagonal element in row x and column y as well as non-diagonal position with row y and column x in correlation matrix.

Step 7: Formation of sets of unipolar orthogonal codes for given values of λ_a

and λ_c such that $1 \leq \lambda_a, \lambda_c \leq w-1$. The upper bound Z of the set of unipolar orthogonal codes with given values of auto-correlation and cross-correlation constraints can be calculated by Johnson bound A [25],[34],[78],[122].



$$Z = \left\lfloor \frac{(n-1)(n-2)\dots(n-\lambda)}{w(w-1)\dots(w-\lambda)} \right\rfloor, \text{ here } \lambda = \max(\lambda_a, \lambda_c)$$

Now, all those codes are selected for which diagonal entries are $\leq \lambda_a$. All the elements of rows and columns, which are not selected, are removed from the correlation matrix, giving a reduced correlation matrix. Within these codes, only those sets of codes with upper bound Z, are selected which has cross-correlation constraints $\leq \lambda_c$ by following method.

- (i) From the reduced correlation matrix only those rows and columns are selected whose numbers of cross-correlation entries with $\leq \lambda_c$ are greater than the upper bound Z of the sets of codes to be generated.
- (ii) In this reduced correlation matrix, number of rows or columns are equal to M. Out of these M codes, all possible combinations of sets of non repeated Z codes are formed mentioning their code numbers. These possible combinations of sets are equal to
$$G = {}^M C_Z = \frac{M(M-1)\dots(M-Z+1)}{Z(Z-1)\dots 2.1}$$
- (iii) Each such set of codes are checked for their maximum cross-correlation constraint $\leq \lambda_c$ through the use of cross-correlation entries from reduced correlation matrix. It will achieve final sets of codes as required.

3.4.2 Computational Complexity of Algorithm - one

The computational complexity of the proposed algorithm - one for the formation of one dimensional unipolar (optical) orthogonal codes is summarized here in the following steps.

I. Calculation for upper bound of the set of one dimensional unipolar (optical) orthogonal codes for code length n, code weight w with auto-correlation and cross-correlation constraint of the set equal to w-1. This upper bound is equal to Johnson bound A for the set given in lemma 2.3.15.

The computational complexity of this step is $O(nw)$.

II. Formation of all one dimensional unipolar (optical) orthogonal codes of code length n, code weight w with auto-correlation and cross-correlation constraint less than or equal to w-1 in standard difference of positions representation (DoPR). The computational complexity of this step is $O(n^{w-1})$.

III. Conversion of every code formed in standard DoPR to extended DoP matrix representation.

The computational complexity of this step is $O(rw^2)$.

IV. Calculation of auto-correlation constraint of each code formed at step II from its EDOP matrix representation as in step III. These values of auto-correlation constraints are put at the position of diagonal elements in correlation matrix $[r \times r]$.

The computational complexity of this step is $O(rw^3)$.



V. Calculation of cross-correlation constraint of every pair of these codes in EDoP matrix representation and putting them in correlation matrix $[r \times r]$ at non diagonal positions. The computational complexity of this step $O(r^2 w^3)$.

VI. Calculation for upper bound or Johnson bound of the set of one dimensional unipolar (optical) orthogonal codes for code length n , code weight w with correlation constraint λ which is maximum of given auto-correlation and cross-correlation constraint. The computational complexity of this step is $O(n\lambda)$.

VII. Formation of reduced correlation matrix whose diagonal elements are always less than or equal to given auto-correlation constraint λ_a and non-diagonal elements are either less than or greater than or equal to cross-correlation constraint λ_c . The computational complexity of this step is $O(r^2)$.

VIII. Formation of all sets of 1-D U(O)OC with maximum cardinality as calculated in step VI, and checking each set for cross-correlation constraint less than or equal to given cross-correlation constraint value with help of reduced correlation matrix.

The computational complexity of this step is $O(r^3)$, where

$$r = \frac{(n-1)(n-2)\dots(n-w+1)}{w(w-1)(w-2)\dots 2.1} \approx \left(\frac{n}{w}\right)^w.$$

The overall computational complexity of the proposed algorithm is of the higher order of $O(r^3)$ which is equivalent $O\left(\left(\frac{n}{w}\right)^{3w}\right)$ which may be polynomial type for $w \ll n$.

3.4.3 Design of Sets of 1-DUOC (Algorithm – two)

The algorithm two is an extended version of algorithm one. In algorithm one the formation of correlation matrix ($N \times N$) is much complex for higher N so that N can not take the values greater than 100. The formation of code sets from the given correlation matrix ($N \times N$) is also much complex. It can be reduced by following algorithm two as given below.

Step 1: same as algorithm one (input code parameters $(n, w, \lambda_a, \lambda_c)$)

Step 2: same as algorithm one (initializing parameters)

Step 3: same as algorithm one (generation of all the N codes in sequence in DoPR)

Step 4: same as algorithm one (calculation of auto-correlation constraint of each of N codes generated at step 3)

Step 5: Take one code C_1 out of all N codes such that maximum non-zero shift auto-correlation of code C_1 is less than or equal to auto-correlation constraint λ_a of desired sets as input in step one. Calculate cross-correlation of pair of codes formed with other $N-1$ codes such that in each pair one code is C_1 . Out of $N-1$ pair of codes only N_1 codes pairing with C_1 are selected which have cross correlation less than or equal to cross correlation constraint λ_c .

Step 6: Repeat step 5 for code C_2 out of all N_1 codes. Get N_2 codes pairing with C_2 out of (N_1-1) pair of codes. The step 6 is repeated till the C_{Z-1} . Where Z is defined and given as maximum



number of codes in the code set formed for given code parameters $(n, w, \lambda_a, \lambda_c)$ such that $(C_1, C_2, \dots, C_{z-1})$ have cross-correlation constraint less than or equal to λ_c . There are total N_{z-1} code which have their cross-correlation value with code C_{z-1} less than or equal to λ_c . Each of these N_{z-1} codes may be treated as code C_z so that there are N_{z-1} set of codes may be formed as $(C_1, C_2, \dots, C_{z-1}, C_z)$

Step 7: The step 6 may be repeated for all possible other codes C_1 to C_{z-1} which are not employed in last steps to get different set of codes following correlation properties.

3.4.4 : Computational Complexity of Algorithm – two

The computational complexity of step 1 to step 7 is remain same as algorithm – one but the value of r is changed for given auto-correlation λ_a and cross-correlation constraint λ_c .

$$\lambda = \max(\lambda_a, \lambda_c)$$

$$r = \frac{(n-1)(n-2)\dots(n-\lambda)}{w(w-1)(w-2)\dots(w-\lambda)} \approx \left(\frac{n}{w}\right)^\lambda$$

The overall computational complexity of the proposed algorithm - two is of the higher order of $O(r^3)$ which is equivalent $O\left(\left(\frac{n}{w}\right)^{3\lambda}\right)$ which may be polynomial type for $w \ll n$ but less complex than algorithm - one.

3.5 Comparison with Ideal Scheme

Both the algorithms proposed here for generation of one dimensional unipolar (optical) orthogonal codes can be compared with an ideal scheme supposed already in chapter – 2 for comparison purpose. The comparison with ideal scheme provides a level of closeness with ideal scheme for generation of one dimensional unipolar orthogonal codes. In following table 3.1 these schemes are compared with an ideal scheme.



OOO(1 D)	Code length 'n'	Weight 'w'	Auto- correlation constraint λ_a	Cross- correlation constraint λ_c	Cardin ality of code- set	No. of code sets	Computati onal complexit y	Other comments
Algorit hm - One	n>0	0<w<n	1 to w-1	1 to w-1	Z	Some random sets	$O\left(\left(\frac{n}{w}\right)^{3w}\right)$	co-prime (w,n)
Algorit hm - Two	n>0	0<w<n	1 to w-1	1 to w-1	Z	Some random sets	$O\left(\left(\frac{n}{w}\right)^{3\lambda}\right)$	co-prime (w,n)
Ideal Scheme	n>0	0<w<n	1 to w-1	1 to w-1	Z	All possible	$\ll O(n^w)$	This scheme is not in existence

Table 3.1: Comparison of proposed algorithms with ideal scheme for generating 1-D U(O)OCs

3.6 Conclusion

The proposed algorithms are able to generate some random sets of one dimensional unipolar orthogonal codes for given code length 'n', code weight 'w' and auto-correlation constraints lying from 1 to w-1. The ideal scheme should generate all possible sets but with very low computational complexity. These proposed schemes are very close to ideal scheme but lack only in the case of computational complexity which may be improved in future.

The drawback of one dimensional unipolar (optical) orthogonal codes are requirement of higher temporal length or code length for high cardinality of code set and increase in computational complexity of algorithm for unipolar (optical) orthogonal code of high code length and code weight. The temporal length of code may be decreased upto much extent with higher cardinality and lower computational complexity of algorithm for designing of matrix orthogonal codes in place of one dimensional unipolar (optical) orthogonal codes. Hence, the next chapter deals mainly with matrix orthogonal codes or two dimensional optical orthogonal codes with their conventional representation, conventional methods to calculate correlation constraints and some already proposed schemes to design matrix orthogonal codes as well as their comparison with an assumed ideal scheme.



CHAPTER 4

4. TWO DIMENSIONAL OPTICAL ORTHOGONAL CODES (2-D OOC)

4.1 Introduction

The two dimensional optical orthogonal code or pseudo orthogonal codes play an important role in terms of better performance than one dimensional unipolar orthogonal code or pseudo orthogonal codes [90]. When one dimensional unipolar orthogonal codes [148-149] are used in Optical CDMA system, the one dimension can be temporal or spectral or spatially placed optical pulses at the position of bit '1's. When the two dimensional orthogonal codes are used any two dimensions can be considered e.g., temporal – spectral (wavelength), temporal – spatial, or spatial – spectral. The optical pulses are placed at the position of bit '1's of the orthogonal code in the two dimensional plane. Two dimensional pseudo orthogonal codes are also called matrix codes. The two dimensional or matrix orthogonal code can be defined with the help of array $(L \times N)$ of the family of $(0,1)$ of constant weight size w with the maximum autocorrelation side-lobe and cross-correlation function be no more than λ_a and λ_c respectively as defined in section 4.3.

4.2 Conventional Representation of 2-D OOC or Matrix Orthogonal Codes

It is known that matrix orthogonal codes is a matrix $(L \times N)$ of binary elements $(0,1)$ in each row and column with weight w , i.e. total number of bit 1's are w in the matrix.

Definition 4.2.1: [1]

2-D Unipolar (optical) orthogonal code or matrix orthogonal code remain same for every column-wise circular shifting.

Proof:

In matrix orthogonal code with matrix size $(L \times N)$, 'L' is the spectral length and 'N' is temporal length of the code. The unipolar (orthogonal) codes are used for multiple asynchronous access of channel. For asynchronous application, the matrix code are defined and remain same for every temporal unit or column-wise circular shifting.

Lemma 4.2.2: [1, 106, 126, 127, 129]

In weighted positions representation (WPR) of matrix orthogonal code, the position of weighted bit is represented by $(a'b)$, where 'a' is row number and 'b' is column number of weighted bit, $1 \leq a \leq L$, $0 \leq b \leq N-1$.



Example 4.2.2 (a):

For $L=4$, $N=5$, weight $w = 4$, suppose the code is

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

Its weighted position representation WPR (1'0, 3'0, 4'1, 2'2)

Lemma 4.2.3: [1, 106, 126, 127, 129]

There are N representations for same matrix orthogonal code in weighted positions representation (WPR).

Proof:

There are ' n ' columns in the matrix orthogonal code. The code remain same on every column-wise circular shifting but not the positions of bit 1's. There are such N matrices representing same code. In weighted positions representation (WPR), a matrix code is represented by positions of bit 1's in the matrix.

Example 4.2.3(a):

The two dimensional code with $L=4$, $N=5$, $w=4$, WPR (1'0, 3'0, 4'1, 2'2) has 5 representations as given below with WPRs after every column wise circular shifting of the matrix code

$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}$
 $\begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$
 $\begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$
 $\begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$

(1'0, 3'0, 4'1, 2'2), (4'0, 2'1, 1'4, 3'4), (2'0, 1'3, 3'3, 4'4), (1'2, 3'2, 4'3, 2'4)

$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$

(1'1, 3'1, 4'2, 2'3)



4.3 Conventional Method for Calculation of Correlation Constraints of 2-D OOC or Matrix Orthogonal Codes and Upper bound of the Set of Codes.

Suppose the Binary matrix codes X and Y belong to the same orthogonal code set $C(L \times N, w, \lambda_a, \lambda_c)$.

Definition 4.3.1:

The auto-correlation constraint of the matrix orthogonal code is the maximum number of overlapping of weighted bits of matrix code with its non-zero column-wise circular shifted versions [1], [133].

Let us take the matrix codes X and Y from same set with code parameters $(L \times N, w, \lambda_a, \lambda_c)$. The maximum non-zero shift auto-correlation or auto-correlation constraint λ_a of the code X is defined and given as follows.

$$\sum_{i=0}^{L-1} \sum_{j=0}^{N-1} x_{i,j} x_{i,j \oplus \tau} \leq \lambda_a, \text{ for } 0 < \tau \leq N-1,$$

Definition 4.3.2:

The cross-correlation constraint for the pair of matrix orthogonal code is the maximum number of overlapping of weighted bits of one matrix code with second matrix code or non-zero column-wise circular shifted versions of the second matrix code [1], [133].

The cross-correlation constraint λ_c for the pair of codes X and Y is defined and given as follows.

$$\sum_{i=0}^{L-1} \sum_{j=0}^{N-1} x_{i,j} y_{i,j \oplus \tau} \leq \lambda_c, \text{ for } 0 \leq \tau \leq N-1.$$

Lemma 4.3.3:

For $\lambda_a = \lambda_c = \lambda$ where $1 \leq \lambda \leq w-1$, The maximum number of two dimensional unipolar (optical) orthogonal codes, Z, in one set is given by the following Johnson bounds [133].

Johnson's bound A is,

$$Z(L \times N, w, \lambda) \leq \left\lfloor \frac{L}{w} \left\lfloor \frac{LN-1}{w-1} \dots \dots \left\lfloor \frac{LN-\lambda}{w-\lambda} \right\rfloor \right\rfloor \right\rfloor = J_A(L \times N, w, \lambda);$$

The Johnson bound B is given conditionally for $w^2 > LN\lambda$

$$Z(L \times N, w, \lambda) \leq \text{Min} \left(L, \left\lfloor \frac{L(w-\lambda)}{w^2 - LN\lambda} \right\rfloor \right) = J_B(L \times N, w, \lambda);$$



The improved Johnson's Bound C is given for any integer k , $1 \leq k \leq \lambda - 1$ such that $(w - k)^2 > (LN - k)(\lambda - k)$, is given as

$$Z(L \times N, w, \lambda) \leq \left\lfloor \frac{L}{w} \left\lfloor \frac{LN - 1}{w - 1} \dots \left\lfloor \frac{LN - (k - 1)}{w - (k - 1)} h \right\rfloor \right\rfloor \right\rfloor = J_c(L \times N, w, \lambda);$$

$$\text{Where } h = \min \left(LN - k, \left\lfloor \frac{(LN - k)(w - \lambda)}{(w - k)^2 - (LN - k)(\lambda - k)} \right\rfloor \right);$$

λ is also called Maximum Collision Parameter (MCP) telling about maximum collisions of array elements bit '1's between any two matrix code words.

There are so many construction schemes for the generation of 2-D OOCs are proposed in different Literatures [80, 106, 111, 113, 120, 126, 129, 132, 133,]. These schemes are capable to design one and more than one set of matrix orthogonal codes with code size less than maximum cardinality given by Johnson bounds in lemma 4.3.3. Some of these described in following for purpose of comparison with ideal scheme and proposed one to design two dimensional unipolar (optical) orthogonal codes.

4.4 2-D OOC Design Schemes Already Proposed in Literature

4.4.1 Temporal / Spatial Addition Modulo L_T (T/S AML) codes Multi-Wavelength OOCs

In [80] the author has provided a very simple scheme for the design of 2-dimensional optical orthogonal codes. The codes are generated on the basis of At Most – One Pulse Per Time (AM-OPPT). Suppose the code C_j is represented by a matrix with L_T rows and non-zero columns with one weighted position for each row. The weighted position is given by R_{ij} and the

$$\text{code } C_j \text{ as } C_j = \begin{bmatrix} R_{0j} \\ R_{1j} \\ R_{2j} \\ \vdots \\ R_{(L_T-1)j} \end{bmatrix}$$

with $R_{0j} = 0$ for $j = 0 : L_T - 1$; and $R_{ij} = (R_{(i-1)j} + j) \bmod (L_T)$ for $1 \leq i \leq L_T - 1$. If L_T is a composite number, the weight $w = L_T$, $\lambda_a = 0$, $\lambda_c = 1$, and the number of codes generated is equal to smallest prime factor of L_T . While if L_T is a prime number, the weight $w = L_T$, $\lambda_a = 0$, $\lambda_c = 1$, and the number of codes generated is equal to L_T . For example let us suppose $L_T = 5$ and non-zero columns is also 5 and the weight $w = L_T = 5$, then 5 codes are constructed as follows



$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

4.4.2 Construction of $(mn, \lambda+2, \lambda)$ Multi-Wavelength OOCs

As described in [106], $(mn, \lambda+2, \lambda)$ represents the set of wavelength–time matrix codes with m rows, n column, weight $w = \lambda+2$ and λ is maximum of auto-correlation constraint λ_a and cross-correlation constraint λ_c . These codes can be constructed by using one dimensional code set $(m, \lambda+2, \lambda)$ and $(n, \lambda+2, \lambda)$ as follows.

The number of code words in the code set $(m, \lambda+2, \lambda)$ is given as s , where

$$s = \frac{(m-1)(m-2)(m-3)\dots(m-\lambda)}{(\lambda+2)!}$$

For which the set of blocks of positional weight obtained from the optical orthogonal codes, can be given as

$$(a_{q0}, a_{q1}, \dots, a_{q(\lambda+1)}) \text{ for } 0 \leq q \leq s-1$$

Similarly the number of code words in the set $(n, \lambda+2, \lambda)$ are given by t , where t is

$$t = \frac{(n-1)(n-2)\dots(n-\lambda)}{(\lambda+2)!}$$

For which the set of blocks of positional weight obtained from the optical orthogonal codes, can be given as

$$(b_{r0}, b_{r1}, \dots, b_{r(\lambda+1)}) \text{ for } 0 \leq r \leq t-1.$$

The positional block code set $(a_{q0}, a_{q1}, \dots, a_{q(\lambda+1)})$ for $0 \leq q \leq s-1$ has $(\lambda+2)!$



distinct permutations represented by $(a_{k,q0}, a_{k,q1}, \dots, a_{k,q(\lambda+1)})$ for $0 \leq q \leq s-1$ and $0 \leq k \leq (\lambda+2)!-1$

The MW- OOC sets can be constructed as follows

$$C_0 = \left\{ (a_{k,q0} \oplus l, b_{r0}), (a_{k,q1} \oplus l, b_{r1}), \dots, (a_{k,q(\lambda+1)} \oplus l, b_{r(\lambda+1)}) \right\}$$

$$C_1 = \left\{ (a_{k,q0} \oplus l, 0), (a_{k,q1} \oplus l, 0), \dots, (a_{k,q(\lambda+1)} \oplus l, 0) \right\}$$

$$C_2 = \left\{ (l, b_{r0}), (l, b_{r1}), \dots, (l, b_{r(\lambda+1)}) \right\}$$

Where $0 \leq k \leq (\lambda+2)!-1$, $0 \leq q \leq s-1$, $0 \leq r \leq t-1$, and $0 \leq l \leq m-1$, \oplus represents modulo m addition. The cardinality C or number of code generated from above set is given as

$$C = mst(\lambda+2)! + ms + mt$$

For example $m=7$, $n=13$, $\lambda=1$, $w=\lambda+2=3$. The number of code words in code set (7,3,1) is $s=1$ and the code word is (0,1,3) as positional weight. The number of code words in the code set (13,3,1) are $t=2$ and code words are { (0,1,4), (0,2,7) } as positional weight.

$(a_{q0}, a_{q1}, a_{q2}) = (0,1,3)$ for $q=0$ and $(b_{r0}, b_{r1}, b_{r2}) = (0,1,4)$ for $r=0$, $(b_{r0}, b_{r1}, b_{r2}) = (0,2,7)$ for $r=1$. (0,1,3) has 6 different permutations given as following

{(0,1,3), (0,3,1), (1,0,3), (1,3,0), (3,0,1), (3,1,0)} for $k=0$ to 5 which is represented with $(a_{k,q0}, a_{k,q1}, a_{k,q2})$ for $q=0$.

For $l=0$ to 6 the code set

$$C_0 = \left\{ (a_{k,q0} \oplus l, 0), (a_{k,q1} \oplus l, 1), (a_{k,q2} \oplus l, 4) \right\} \text{ for } r=0,$$

$$C'_0 = \left\{ (a_{k,q0} \oplus l, 0), (a_{k,q1} \oplus l, 2), (a_{k,q2} \oplus l, 7) \right\} \text{ for } r=1,$$

$$C_1 = \left\{ (a_{0,q0} \oplus l, 0), (a_{0,q1} \oplus l, 0), (a_{0,q2} \oplus l, 0) \right\}$$

$$C_2 = \left\{ (l, 0), (l, 1), (l, 4) \right\} \text{ for } r=0,$$

$$C'_2 = \left\{ (l, 0), (l, 2), (l, 7) \right\} \text{ for } r=1,$$

Total possible code words in this example are $7 \times 6 + 7 \times 6 + 7 \times 1 + 7 \times 1 + 7 \times 1 = 105$.



4.4.3 2D- matrix codes from spanning ruler or optimum Golomb ruler

A spanning ruler or optimum Golomb ruler [111], is a binary (0,1) sequence of length n such that the distance between any two weighted bits (i.e. bit '1') is non-repetitive. The optimum Golomb ruler sequence can generate other sequences M_1 to M_p by introducing $(p-1)$ zeros in the right and cyclically right shifting 0 to $p-1$ times making all the sequence of same length. The length of sequence ' n ' could be break into two integer factors x and y such that $n = xy$. x and y may take different values of integers to generate different matrix codes of x rows and y columns.

It can be best understood by following example as in [111]. Suppose an optimum Golomb sequence g_1 of length 26 and weight $w=7$ is given as $g_1 = [11001000001000000010000101]$ linearly right shifting the sequence and making it of length 32, the following M_i sequences are generated

$$\begin{aligned} M_1 &= [11001000001000000010000101000000] \\ M_2 &= [01100100000100000001000010100000] \\ M_3 &= [00110010000010000000100001010000] \\ M_4 &= [00011001000001000000010000101000] \\ M_5 &= [00001100100000100000001000010100] \\ M_6 &= [00000110010000010000000100001010] \\ M_7 &= [00000011001000001000000010000101] \end{aligned}$$

In matrix form the sequences M_1 to M_7 can be written in column wise taking 4 - 4 elements in the columns as

$$\begin{aligned} M_1 &= \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}, & M_2 &= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}, \\ M_3 &= \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}, & M_4 &= \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \\ M_5 &= \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}, & M_6 &= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}, \end{aligned}$$



$$M_7 = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Here by periodic shifting (column wise) we can conclude that $M_5 = M_1$; $M_6 = M_2$; $M_7 = M_3$; Hence only 4 optical orthogonal codes M_1, M_2, M_3, M_4 can be constructed.

4.4.4 2D-wavelength/time OOCs based on Balanced Codes for Differential Detection (BCDD) and antipodal signaling

In antipodal signaling of binary signal, the bit '1' or bit '0' takes same amount of power for it's transmission. The spreading sequence of bit '1' and bit '0' are chosen as s_u^1 and s_u^0 respectively such that inner product of s_u^1 and s_u^0 is zero while inner product of s_u^1 or s_u^0 with itself becomes equal to hamming weight w of the codes. The 2D matrix code s_u^1 and s_u^0 of size mn , consists of equal number of '1's and '0's with weight $w = \frac{1}{2} mn$. The inner product of a and b is defined as $\langle a, b \rangle = \sum_{i=1}^m \sum_{j=1}^n a_{ij} b_{ij}$; $a_{ij}, b_{ij} \in (0,1)$

a_{ij} denotes the binary element of the matrix code at i^{th} row and j^{th} column. The transmitted signal by user u is $t_u = b_u(s_u^1, \tau_u) + \bar{b}_u(s_u^0, \tau_u)$, where $b_u \in (0,1)$, $\bar{b}_u = b_u - 1$, $s_u^1 = 1 - s_u^0$ and τ_u is the asynchronous delay introduced by user u . The received multiplexed signal r is given as

$r = \sum_{k=1}^N b_k(s_k^1, \tau_k + \tau) + \bar{b}_k(s_k^0, \tau_k + \tau)$. Where τ is the channel delay whose effect can be eliminated by synchronizing the transmitted and received chip sequences.

At the receiver for user u , the output obtained after matched filtering and differential detection is $r_u = (\langle r, s_u^1 \rangle + \eta_1) - (\langle r, s_u^0 \rangle + \eta_2)$ where η_1 and η_2 are the noises added during balanced differential detection. These noises can be assumed negligible in comparison to multiple access interference and

$r_u = (b_u^1 - b_u^0)w + \left\langle \sum_{k=1, k \neq u}^N b_k(s_k^1, \tau_k + \tau) + \bar{b}_k(s_k^0, \tau_k + \tau), (s_u^1 - s_u^0) \right\rangle$. Here the first term represents the detected output of user u and second term represents the MAI due to presence of unwanted signals of other users.

The BER of user u can be calculated as

$$(BER)_u = \Pr(b_u=1) \cdot \Pr(MAI < -w) + \Pr(b_u=0) \cdot \Pr(MAI > w)$$

The codes can be designed with mentioned constraints as



1) The hamming weight of the code is half of matrix code size m into n .
 $\langle s_u^1, s_u^1 \rangle = \langle s_u^0, s_u^0 \rangle = w = mn/2$.

2) The spreading sequence s_u^1 for bit '1' is just complement of spreading sequence s_u^0 for bit '0' such that $s_u^0 = 1 - s_u^1$.

3) The auto-correlation of s_u^1 or s_u^0 with it's column wise cyclically shifted sequence is at most λ_a

$$\bar{b}_u \langle (s_u^0, \tau_u), s_u^1 \rangle - b_u \langle (s_u^1, \tau_u), s_u^0 \rangle \leq \lambda_a$$

4) The cross-correlation of s_u^1 and s_u^0 with the spreading sequence s_v^1 and s_v^0 of other user is at most λ_c

$$\langle (b_v(s_v^1, \tau_v) + \bar{b}_v(s_v^0, \tau_v)), (s_u^1 - s_u^0) \rangle \leq \lambda_c$$

Here in designing of codes λ_a is considered equal to λ_c

i.e. $\lambda_a = \lambda_c$ by using greedy algorithm the codes are constructed with above conditions such that next generated code is found under mentioned conditions until no new codes are found [113].

4.4.5 2D-wavelength/time OOCs based on Carrier Hopping Prime Code

The Carrier Hopping Prime Code (CHPC) set is particular type of 2D OOCs ($L \times N$, w , λ_a , λ_c) with $L=w$, $N = p_1 p_2 \dots p_k$, $\lambda_a=0$ and $\lambda_c=1$. Here $p_1, p_2, p_3, \dots, p_k$ are prime numbers such that $p_k \geq p_{k-1} \geq \dots \geq p_2 \geq p_1$ and $w=p_1$. The codes are constructed with ordered pair as follows

$$\begin{aligned} & \{[(0, 0), (1, i_1 + i_2 p_1 + i_3 p_1 p_2 + \dots + i_k p_1 p_2 \dots p_k), \\ & (2, 2 \otimes_{p_1} i_1 + (2 \otimes_{p_2} i_2) p_1 + (2 \otimes_{p_3} i_3) p_1 p_2 + \dots + (2 \otimes_{p_k} i_k) p_1 p_2 \dots p_{k-1}), \dots \\ & \dots, (p_1 - 1, (p_1 - 1) \otimes_{p_1} i_1 + ((p_1 - 1) \otimes_{p_2} i_2) p_1 + ((p_1 - 1) \otimes_{p_3} i_3) p_1 p_2 + \dots \\ & \dots + ((p_1 - 1) \otimes_{p_k} i_k) p_1 p_2 \dots p_{k-1})] : \\ & i_1 \in [0, p_1 - 1], i_2 \in [0, p_2 - 1], \dots, i_k \in [0, p_k - 1] \} \end{aligned}$$

Where \otimes_{p_j} denotes a modulo- p_j multiplication for $j = \{1, 2, 3, \dots, k\}$, resulting in $p_1 p_2 \dots p_k$ or equal to n number of orthogonal matrices. While for the case $L = w.p' \leq p_1$, where p' is a prime number, $N = p_1 p_2 \dots p_k$, $w = p_1$, resulting in Np' number of orthogonal matrices for $\lambda_a=0$ and $\lambda_c=1$, the Extended Carrier Hopping Prime Code (ECHPC) can be constructed by replacing the first element (0,0) in each ordered pair in above equation by



$\{[l_1, (l_1 \oplus_w l_2) + w, (l_1 \oplus_w \otimes_w (2 \otimes_p l_2)) + 2w, \dots,$
 $(l_1 \oplus_w ((w-1) \otimes_p l_2)) + (w-1)w]: l_1 \in [0, w-1], l_2 \in [0, w-1]\}$
and $\{[l_2w, l_2w+1, \dots, l_2w+w-1]: l_2 \in [0, p'-1]\}$ respectively.

Where \oplus_w denotes a modulo- w addition. It results in $(L \times N, w, 0, 1)$ set of matrix orthogonal codes with $(w^2 + p')N$ number of matrices [126, 127, 129].

4.4.6 Multiple wavelength OOCs under prime sequence permutations

Suppose a Galois Field $GF(p) = (0, 1, 2, \dots, p-1)$, p is a prime, can be used to construct the prime sequence $S_x^p = \{s_x^p(0), s_x^p(1), s_x^p(2), \dots, s_x^p(p-1)\}$, $s_x^p(j) = x \cdot j \pmod{p}$ for $x, j \in GF(p)$; For $GF(7) = (0, 1, 2, 3, 4, 5, 6)$, the prime sequences

$$s_x^p(0) = (0, 0, 0, 0, 0, 0, 0);$$

$$s_x^p(1) = (0, 1, 2, 3, 4, 5, 6);$$

$$s_x^p(2) = (0, 2, 4, 6, 1, 3, 5);$$

$$s_x^p(3) = (0, 3, 6, 2, 5, 1, 4);$$

$$s_x^p(4) = (0, 4, 1, 5, 2, 6, 3);$$

$$s_x^p(5) = (0, 5, 3, 1, 6, 4, 2);$$

$$s_x^p(6) = (0, 6, 5, 4, 3, 2, 1);$$

The one dimensional optical orthogonal code can be converted into two dimensional OOCs as, suppose the code for $(7, 3, 1, 1)$ is given as (1101000) . Using the prime sequence $s_x^p(0), s_x^p(1), \dots, s_x^p(6)$ with only their first three ($w=3$) weighing positions, the following group G_0, G_1, \dots, G_6 of codes can be constructed for wavelength $\lambda_0, \lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6$ as

$$G_0 = \{[\lambda_0 \lambda_0 0 \lambda_0 0 0 0], [\lambda_1 \lambda_1 0 \lambda_1 0 0 0], [\lambda_2 \lambda_2 0 \lambda_2 0 0 0], [\lambda_3 \lambda_3 0 \lambda_3 0 0 0],$$

$$[\lambda_4 \lambda_4 0 \lambda_4 0 0 0], [\lambda_5 \lambda_5 0 \lambda_5 0 0 0], [\lambda_6 \lambda_6 0 \lambda_6 0 0 0]\}$$

$$G_1 = \{[\lambda_0 \lambda_1 0 \lambda_2 0 0 0], [\lambda_1 \lambda_2 0 \lambda_3 0 0 0], [\lambda_2 \lambda_3 0 \lambda_4 0 0 0], [\lambda_3 \lambda_4 0 \lambda_5 0 0 0],$$

$$[\lambda_4 \lambda_5 0 \lambda_6 0 0 0], [\lambda_5 \lambda_6 0 \lambda_0 0 0 0], [\lambda_6 \lambda_0 0 \lambda_1 0 0 0]\}$$

$$G_2 = \{[\lambda_0 \lambda_2 0 \lambda_4 0 0 0], [\lambda_1 \lambda_3 0 \lambda_5 0 0 0], [\lambda_2 \lambda_4 0 \lambda_6 0 0 0], [\lambda_3 \lambda_5 0 \lambda_0 0 0 0],$$

$$[\lambda_4 \lambda_6 0 \lambda_1 0 0 0], [\lambda_5 \lambda_0 0 \lambda_2 0 0 0], [\lambda_6 \lambda_1 0 \lambda_3 0 0 0]\}$$

$$G_3 = \{[\lambda_0 \lambda_3 0 \lambda_6 0 0 0], [\lambda_1 \lambda_4 0 \lambda_0 0 0 0], [\lambda_2 \lambda_5 0 \lambda_1 0 0 0], [\lambda_3 \lambda_6 0 \lambda_2 0 0 0],$$

$$[\lambda_4 \lambda_0 0 \lambda_3 0 0 0], [\lambda_5 \lambda_1 0 \lambda_4 0 0 0], [\lambda_6 \lambda_2 0 \lambda_5 0 0 0]\}$$

$$G_4 = \{[\lambda_0 \lambda_4 0 \lambda_1 0 0 0], [\lambda_1 \lambda_5 0 \lambda_2 0 0 0], [\lambda_2 \lambda_6 0 \lambda_3 0 0 0], [\lambda_3 \lambda_0 0 \lambda_4 0 0 0],$$

$$[\lambda_4 \lambda_1 0 \lambda_5 0 0 0], [\lambda_5 \lambda_2 0 \lambda_6 0 0 0], [\lambda_6 \lambda_3 0 \lambda_0 0 0 0]\}$$

$$G_5 = \{[\lambda_0 \lambda_5 0 \lambda_3 0 0 0], [\lambda_1 \lambda_6 0 \lambda_4 0 0 0], [\lambda_2 \lambda_0 0 \lambda_5 0 0 0], [\lambda_3 \lambda_1 0 \lambda_6 0 0 0],$$

$$[\lambda_4 \lambda_2 0 \lambda_0 0 0 0], [\lambda_5 \lambda_3 0 \lambda_1 0 0 0], [\lambda_6 \lambda_4 0 \lambda_2 0 0 0]\}$$



$$G_6 = \{[\lambda_0 \lambda_6 0 \lambda_5 0 0 0], [\lambda_1 \lambda_0 0 \lambda_6 0 0 0], [\lambda_2 \lambda_1 0 \lambda_0 0 0 0], [\lambda_3 \lambda_2 0 \lambda_1 0 0 0], \\ [\lambda_4 \lambda_3 0 \lambda_2 0 0 0], [\lambda_5 \lambda_4 0 \lambda_3 0 0 0], [\lambda_6 \lambda_5 0 \lambda_4 0 0 0] \}$$

Here for the code (1101000) of (7, 3, 1, 1), there are maximum 7 x 7 or (p x p) code can be constructed. If one dimensional code size is Z_1 , then maximum $Z_1 p^2$ two dimensional codes can be constructed [120]. The other code groups of different weight (1 to 7) can be generated in the same manner as described above.

4.5 The Comparison with Ideal Scheme

It can be assumed, an ideal scheme generating all possible sets of two dimensional or matrix unipolar (optical) orthogonal codes with maximum cardinality. If the proposed schemes in literature generating two-dimensional optical orthogonal codes are being compared with the assumed ideal scheme, it can be explored that till how much extent the proposed scheme is getting closed to ideal. The comparison table of the proposed scheme in literature with ideal one is given in following Table 4.1.

4.6 Conclusion

In this chapter, it can be summarized that there is need to develop an scheme generating two dimensional optical orthogonal codes closed to assumed ideal scheme. The generated two dimensional optical orthogonal codes can be utilized for assignment of orthogonal codes to distinct users of wavelength-hopping time spreading optical CDMA system. The result of ideal scheme may provide flexibility for selection of two dimensional optical orthogonal codes and even selection of one set out of designed multiple sets of two dimensional optical orthogonal codes to increase the inherent security and spectral efficiency.



OOO (2D)	Code Array $L \times N$	Weight 'w'	Auto-correlation constraint λ_a	Cross-correlation constraint λ_c	Cardinality of set	Number of sets	Computational Complexity	Other comments
T/S AML	$L \times L$	L	0	1	L	one	$\ll O(L^w)$	L is a composite number
Multi-wavelength	$m \times n$	$\lambda + 2$	λ	λ	$ms + mt + mst(\lambda + 2)$	one	$\ll O(n^w)$	s and t no. of 1D OOCs of length prime m and n respectively
Spanning Ruler or Optim. Golomb ruler	$(L \times N) > n$	w	0	1	$(L \times N) - n + 1$	one	$\ll O(N^w)$	n is length of optim. Golomb sequence
BCDD and antipodal signaling	$m \times n$	$w = mn/2$	λ	λ	$< Z$	one	$\ll O(n^w)$	Search method
CHPC	$L = wp'$ $N = p_1 p_2 \dots p_k$	$w = p_1$	0	1	Np'	one	$\ll O(N^w)$	p', p_1, p_2, \dots, p_k are prime
MW OOC Prime Sequence	$p \times p$	$w < p$	1	1	$p \times p$	one	$\ll O(p^w)$	p is a prime
Ideal Scheme	$L \times N$ $L > 0, N > 0$	$1 \leq w \leq LN - 1$	$1 \leq \lambda_a \leq w - 1$	$1 \leq \lambda_c \leq w - 1$	$= Z$	All possible	Very low	Scheme is not in existence

Table 4.1: Comparison of proposed 2-D OOCs design schemes with ideal one.



CHAPTER 5

5. DESIGN OF TWO DIMENSIONAL UNIPOLAR (OPTICAL) ORTHOGONAL CODES (2-D U(O)OC) AND THEIR MAXIMAL CLIQUE SETS

5.1 Introduction

The need of two dimensional unipolar (optical) orthogonal codes over one dimensional unipolar (optical) orthogonal is obvious for lower temporal length, higher cardinality of the set of codes. Within two dimensional unipoar (optical) orthogonal codes, there is a need to represent these codes in unique manner. Conventionally these codes are represented by weighted positions representation giving N representation to same code. There is a need to provide simple method to calculate auto-correlation and cross-correlation constraint in comparison to conventional complex methods for calculation of correlation constraints. There is also a need to develop an algorithm to design multiple sets of two dimensional unipolar (optical) codes with maximum cardinality for known auto-correlation and cross-correlation constraints of the set of codes. In the next section, a unique representation of two dimensional unipolar orthogonal codes named be difference of positions representation (DoPR), is proposed with its characteristics. A new lower complex method for calculation of auto-correlation as well as cross-correlation constraints of one dimensional unipolar (optical) orthogonal codes are also proposed in this chapter. These generated codes provide flexibility for selection of one dimensional unipolar (optical) orthogonal codes from same set to multiple users of wavelength hopping time spreading optical code division multiple access (CDMA) systems. The generated multiple sets provide flexibility for selection of set of one dimensional unipolar orthogonal codes to be assigned to a set of users of wavelength hopping time spreading optical CDMA systems. Two search algorithms are proposed which find multiple sets of unipolar (optical) orthogonal codes through one dimensional unipolar (optical) orthogonal codes and finding their multiple sets have been discussed. The cardinality of each code-set approach the Johnson's bound for different correlation constraints. This newly proposed scheme has also been compared with ideal one which is assumed in chapter four. The first algorithm finds all possible sets of unipolar (optical) orthogonal codes with maximum cardinality for matrix code dimension $(L \times N)$, code weight 'w' such that w and LN are co-prime, auto-correlation constraint and cross-correlation constraint from the range 1 to w-1 using direct search method. This algorithm works well upto $LN = 46$ and $w=4$ for auto-correlation and cross-correlation constraints lying from 1 to 3. The second algorithm uses clique search method to find all sets of codes not only for the same length and the same weight but also for the multi-length and multi-weight one dimensional unipolar orthogonal



codes. This algorithm work well upto $LN= 256$ and $w=5$ for auto-correlation and cross-correlation constraints lying from 1 to 2.

5.2 Difference of Positions Representation (DoPR) of 2-D U(O)OC or Matrix Orthogonal Codes

It is known that matrix orthogonal codes is a matrix ($L \times N$) of binary elements (0,1) in each row and column with weight w , i.e. total number of bit 1's are w in the matrix.

Lemma 5.2.1:

In the matrix orthogonal code, the difference of positions of consecutive weighted columns remain same on every column-wise circular shifting of the code.

Proof:

The weighted column means the column having at least one weighted bit or bit '1'. On every column-wise circular shifting, there is no change in consecutive difference of positions of weighted columns similar to difference of positions of consecutive weighted bits in one dimensional unipolar (optical) orthogonal codes.

Lemma 5.2.2:

The row positions of the weighted bit/bits or the differences of consecutive positions of bit 1's in every column remain same on every column-wise circular shifting of the code.

Proof: straightforward

Lemma 5.2.3:

In DoPR of matrix orthogonal code, the position of weighted bit is represented by $(a'd)$, where 'a' is row number of weighted bit and 'd' is consecutive difference of column position of next weighted bit with position of column of the current weighted bit. $1 \leq a \leq L$, $0 \leq d \leq N-1$. It is a unique representation of code.

Example 5.2.3(a):

Suppose the matrix orthogonal code X with $L=4$, $N=5$, weight $w= 7$,



$$\text{Code X} = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix};$$

$$X(\text{WPR}) = (1'0, 3'0, 2'1, 4'1, 1'4, 3'4, 4'4);$$

$$X(\text{DoPR}) = (1'0, 3'1, 2'0, 4'3, 1'0, 3'0, 4'1).$$

There are N=5 columns. As per lemma 4.2.3, there are 5 WPR of this code are possible which are given as follows with the DoPR of the code.

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix},$$

$$\begin{bmatrix} 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\text{WPR}(1'0, 3'0, 2'1, 4'1, 1'4, 3'4, 4'4),$$

$$\text{DoPR}(1'0, 3'1, 2'0, 4'3, 1'0, 3'0, 4'1),$$

$$\text{WPR}(2'0, 4'0, 1'3, 3'3, 4'3, 1'4, 3'4),$$

$$\text{DoPR}(2'0, 4'3, 1'0, 3'0, 4'1, 1'0, 3'1),$$

$$\begin{bmatrix} 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \end{bmatrix},$$

$$\begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix},$$

$$\text{WPR}(1'2, 3'2, 4'2, 1'3, 3'3, 2'4, 4'4),$$

$$\text{DoPR}(1'0, 3'0, 4'1, 1'0, 3'1, 2'0, 4'3),$$

$$\text{WPR}(1'1, 3'1, 4'1, 1'2, 3'2, 2'3, 4'3),$$

$$\text{DoPR}(1'0, 3'0, 4'1, 1'0, 3'1, 2'0, 4'3),$$

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \end{bmatrix}$$

$$\text{WPR}(1'0, 3'0, 4'0, 1'1, 3'1, 2'2, 4'2),$$

$$\text{DoPR}(1'0, 3'0, 4'1, 1'0, 3'1, 2'0, 4'3).$$

It can be observed in this example that in every column wise circular shifting of the code, WPR of code changed but DoPR remain same, it is only circular shifted versions of DoPR $(1'0, 3'0, 4'1, 1'0, 3'1, 2'0, 4'3)$ without changing the numerical values. Hence it can be claimed that DOPR is a unique representation of two dimensional unipolar (optical) orthogonal codes.



Lemma 5.2.4:

In DoPR of matrix orthogonal code $(a_1'd_1, a_2'd_2, \dots, a_w'd_w)$, $d_1 + d_2 + \dots + d_w = N$, where N is number of columns in binary matrix orthogonal code.

Lemma 5.2.5:

The DoPR of matrix orthogonal code $(a_1'd_1, a_2'd_2, \dots, a_w'd_w)$ may be converted to WPR $(a_1'b_1, a_2'b_2, \dots, a_w'b_w)$ and vice versa with 0th column to be weighted necessarily as follows under modulo N arithmetic,

$$\begin{aligned} b_1 &= 0; \\ b_2 &= b_1 + d_1; \\ b_3 &= b_2 + d_2; \\ &\dots; \\ b_w &= b_{w-1} + d_{w-1}; \end{aligned}$$

5.3 Calculation of Correlation Constraints

Lemma 5.3.1:

The auto-correlation constraint of a matrix orthogonal code is equal to maximum overlapping of weighted positions of any two out of 'N' representations of the code in WPR (lemma 4.2.3).

$$\lambda_a \geq (X_p) \cap (p + X_p), \quad (0 < p \leq N-1)$$

Where X_p represent to WPR of matrix code X and $(p + X_p)$ represent to WPR of p times column wise right circular shifted version of code X.

Example 5.3.1(a):

Suppose matrix code X, with its WPR X_p as

$$X = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}, \quad X_p = \text{WPR}(1'0, 3'0, 2'1, 4'1, 1'4, 3'4, 4'4),$$

$$\begin{aligned} 1 + X_p &= \text{WPR}(1'0, 3'0, 4'0, 1'1, 3'1, 2'2, 4'2), \\ 2 + X_p &= \text{WPR}(1'1, 3'1, 4'1, 1'2, 3'2, 2'3, 4'3), \\ 3 + X_p &= \text{WPR}(1'2, 3'2, 4'2, 1'3, 3'3, 2'4, 4'4), \\ 4 + X_p &= \text{WPR}(2'0, 4'0, 1'3, 3'3, 4'3, 1'4, 3'4), \\ (X_p) \cap (1 + X_p) &= 2 \end{aligned}$$



$$(X_p) \cap (2 + X_p) = 1$$

$$(X_p) \cap (3 + X_p) = 1$$

$$(X_p) \cap (4 + X_p) = 2$$

Hence as per lemma 5.3.1 auto-correlation constraint $\lambda_a = 2$.

Lemma 5.3.2:

The cross-correlation constraint for the pair of matrix orthogonal codes (X,Y) is the maximum overlapping of weighted positions of one matrix code with 'N' representations of other matrix code in WPR.

$$\lambda_c \geq (X_p) \cap (p + Y_p), \quad (0 \leq p \leq N-1)$$

Alternatively

$$\lambda_c \geq (Y_p) \cap (p + X_p), \quad (0 \leq p \leq N-1)$$

Where X_p represent to WPR of matrix code X and $(p + Y_p)$ represent to WPR of p times column wise circular shifted version of code Y and vice versa.

Example 5.3.2(a):

Suppose matrix codes X and Y, with their WPR X_p and Y_p respectively as

$$X = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}, \quad X_p = \text{WPR}(1'0, 3'0, 2'1, 4'1, 1'4, 3'4, 4'4), \quad \lambda_a = 2$$

And

$$Y = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}, \quad Y_p = \text{WPR}(1'0, 2'0, 4'1, 2'2, 3'2, 1'4, 4'4), \quad \lambda_a = 2$$

$$1 + Y_p = \text{WPR}(1'0, 4'0, 1'1, 2'1, 4'2, 2'3, 3'3),$$

$$2 + Y_p = \text{WPR}(1'1, 4'1, 1'2, 2'2, 4'3, 2'4, 3'4),$$

$$3 + Y_p = \text{WPR}(2'0, 3'0, 1'2, 4'2, 1'3, 2'3, 4'4),$$

$$4 + Y_p = \text{WPR}(4'0, 2'1, 3'1, 1'3, 4'3, 1'4, 2'4),$$

$$(X_p) \cap (1 + Y_p) = 2$$

$$(X_p) \cap (2 + Y_p) = 2$$

$$(X_p) \cap (3 + Y_p) = 2$$

$$(X_p) \cap (4 + Y_p) = 2$$

Hence as per lemma 5.3.2 cross-correlation constraint for pair of codes X and Y be $\lambda_c = 2$.



5.4 Formation of the Maximal Sets of 2-DU(O)OC:

The maximal sets of 2-DUOC for fixed code parameters $(L \times N, w, \lambda_a, \lambda_c)$ are formed through following two proposed algorithms.

5.4.1 Algorithm one to design the maximum sets of 2-DUOC

This algorithm can generate multiple sets of two dimensional unipolar orthogonal codes for given matrix code dimensions $(L \times N)$, code weight 'w' and correlation constraints lying from 1 to w-1, such that $w^2 < LN$. The codes are generated in difference of positions representation (DoPR). The steps of algorithm are as following.

Step-1: Input matrix code dimensions L, the number of rows, N, the number of columns, code weight 'w', auto-correlation constraint ' λ_a ' and cross-correlation constraint ' λ_c ' for the code sets to be generated.

Step-2: Initialize w variables $(m_1, m_2, \dots, m_{w-1})$ equal to one and $m_w = (LN - (m_1 + m_2 + \dots + m_{w-1}))$.

Step-3: Generate all the one dimensional codes of set (n, w) with code length $n = LN$, in standardized DoPR in sequence starting from $(1, 1, \dots, n - w + 1)$ to (m_1, m_2, \dots, m_w) with enumeration.

$$(i) m_w \geq (m_1, m_2, \dots, m_{w-1}) \geq 1 \quad (ii) \left\lceil \frac{n}{w} \right\rceil \leq m_w \leq (n - w + 1).$$

The variables $(m_1, m_2, \dots, m_{w-1}, m_w)$ in DoPR, represent to difference of weighted positions or position of bit 1's in serial and circular order in the codes which are generated very similar to 3.4.1, algorithm one to design the maximal sets of 1-DUOC.

Step 4: Conversion of one dimensional unipolar orthogonal codes to two dimensional unipolar orthogonal codes

(i) Conversion of one dimensional code (DoPR) (m_1, m_2, \dots, m_w) into corresponding one dimensional code (WPR) $(1, m_1 + 1, m_1 + m_2 + 1, \dots, 1 + m_1 + m_2 + \dots + m_{w-1})$ [156].

(ii) Conversion of one dimensional code (WPR) into two dimensional code (WPR) by dividing each weighted position by 'L' to get quotient 'b' and remainder 'a' for each weighted position. Here each a'b represent to each weighted position in matrix orthogonal code. 'a' stands for row position and 'b' stands for column position as in lemma 4.2.2.

Lemma 5.4.1.1:

The matrix orthogonal code with a'b weighted positions can be converted into



corresponding binary matrix orthogonal code by putting binary digit '1' at weighted positions and '0' otherwise. This binary matrix orthogonal code can be converted into 'L' binary matrix orthogonal codes by every row wise circular shifting of the code.

(iii) Conversion of two dimensional code (WPR) into two dimensional code (DoPR) by getting difference 'd' of two columns of consecutive weighted positions as obtained in (ii) in circular order so that each weighted position is represented by (a'd) as in Lemma 5.2.5. It will be standard DoPR of two dimensional codes if it is obtained from one dimensional code in standard DoPR.

Lemma 5.4.1.2:

The two dimensional unipolar (optical) orthogonal code (in DoPR) can be converted into binary matrix orthogonal code through intermediate two dimensional unipolar (optical) orthogonal code (in WPR).

Step 5: Calculation of auto-correlation constraints

For the generated 2 dimensional codes in step 4, the auto-correlation constraint of each code can be calculated through the use of proposed method for calculation of correlation constraints given in lemma 5.3.1.

Step 6: Calculation of cross-correlation constraints

The cross-correlation constraint for each pair of two dimensional unipolar orthogonal codes generated in step-4 is calculated through the use of proposed method described earlier in lemma 5.3.2. The cross-correlation for each pair containing code#1 with code of code number greater than 1, secondly the code#2 with code of code number greater than 2, upto code#(M-1) with code#M.

Step 7: Formation of correlation matrix

In step 4, the number of generated codes are M. A $M \times M$ matrix can be formed in such a way that it contains correlation of code# x with code# y, for $1 \leq (x, y) \leq M$.

When $x = y$, it represent to maximum auto-correlation for non zero shift or auto-correlation constraint of code# x or code# y, which form diagonal elements of $M \times M$ correlation matrix. For $x \neq y$, cross correlation constraint of code# x with code# y is found as a non-diagonal element in row x and column y as well as non-diagonal position with row y and column x in correlation matrix.

Step 8: Formation of sets of unipolar orthogonal codes for given values of λ_a and λ_c such that $1 \leq \lambda_a, \lambda_c \leq w-1$. For given values of auto-correlation and cross-correlation constraints for the set of unipolar orthogonal codes, the upper bound Z of such set of codes can be calculated by Johnson bound A [122].

$$Z = \left\lfloor L \frac{(LN-1)(LN-2)\dots(LN-\lambda)}{w(w-1)\dots(w-\lambda)} \right\rfloor, \text{ here } \lambda = \max(\lambda_a, \lambda_c)$$

Now, all those codes are selected for which diagonal entries are $\leq \lambda_a$. All the rows and column for the codes which not selected are removed from the correlation matrix, giving a reduced



correlation matrix. Within these codes, only those sets of codes with upper bound Z , are selected which has cross-correlation constraints $\leq \lambda_c$ by following method.

- (iv) From the reduced correlation matrix only those rows and columns are selected whose number of cross-correlation entries with $\leq \lambda_c$ are greater than the upper bound Z of the sets of codes to be generated.
- (v) In this reduced correlation matrix, number of rows or columns are equal to P . Out of these P codes, all possible combinations of sets of non repeated Z codes are formed mentioning their code numbers. These possible combinations of sets are equal to
$$G = {}^P C_Z = \frac{P(P-1)\dots(P-Z+1)}{Z(Z-1)\dots 2.1}$$
- (vi) Each such set of codes are checked for their maximum cross-correlation constraint $\leq \lambda_c$ through the use of cross-correlation entries from reduced correlation matrix. It will achieve final sets of codes as required.

5.4.2 Computational Complexity of Algorithm - one

The computational complexity of the proposed algorithm for the formation of two dimensional unipolar (optical) orthogonal codes is summarized here in the following steps.

I. Calculation for upper bound of the set of one dimensional unipolar (optical) orthogonal codes for code length n , code weight w with auto-correlation and cross-correlation constraint of the set equal to $w-1$. This upper bound is equal to Johnson bound A for the set given in lemma 2.3.15.

The computational complexity of this step is $O(LNw)$.

II. Formation of all two dimensional unipolar (optical) orthogonal codes of code dimensions $(L \times N)$, code weight w with auto-correlation and cross-correlation constraint less than or equal to $w-1$ in standard difference of positions representation (DoPR). The computational complexity of this step is $O((LN)^{w-1})$.

III. Conversion of one dimensional codes to two dimensional or matrix orthogonal codes. The computational complexity of this step is $O(rw^2)$.

IV. Calculation of auto-correlation constraint of each code formed at step 4 form. These values of auto-correlation constraints are put at the position of diagonal elements in correlation matrix $[r \times r]$.

The computational complexity of this step is $O(rw^3)$.

V. Calculation of cross-correlation constraint of every pair of these codes and putting them in correlation matrix $[r \times r]$ at non diagonal positions. The computational complexity of this step $O(r^2w^3)$.

VI. Calculation for upper bound or Johnson bound of the set of two dimensional unipolar (optical) orthogonal codes for code dimension $(L \times N)$, code weight ' w ' with correlation constraint



λ which is maximum of given auto-correlation and cross-correlation constraint. The computational complexity of this step is $O(LN\lambda)$.

VII. Formation of reduced correlation matrix whose diagonal elements are always less than or equal to given auto-correlation constraint λ_a and non-diagonal elements are either less than or greater than or equal to cross-correlation constraint λ_c . The computational complexity of this step is $O(r^2)$.

VIII. Formation of all sets of 2-D U(O)OC with maximum cardinality as calculated in step VI, and checking each set for cross-correlation constraint less than or equal to given cross-correlation constraint value with help of reduced correlation matrix.

The computational complexity of this step is $O(r^3)$, where

$$r = \frac{(LN-1)(LN-2)\dots(LN-w+1)}{w(w-1)(w-2)\dots 2.1} \approx \left(\frac{LN}{w}\right)^w.$$

The overall computational complexity of the proposed algorithm is of the higher order of $O(r^3)$ which is equivalent $O\left(\left(\frac{LN}{w}\right)^{3w}\right)$ which may be polynomial type for $w \ll LN$.

5.4.3 Design of Sets of 2-DUOC (Algorithm – two)

The algorithm two is an extended version of algorithm one. In algorithm one the formation of correlation matrix (MxM) is much complex for higher M. The formation of code sets from the given correlation matrix (MxM) is also much complex. It can be reduced by following algorithm two as given below.

Step 1: same as algorithm one (input code parameters $(L \times N, w, \lambda_a, \lambda_c)$)

Step 2: same as algorithm one (initializing parameters)

Step 3: same as algorithm one (generation of all the M codes in sequence in DoPR)

Step 4: same as algorithm one (calculation of auto-correlation constraint of each of M codes generated at step 3)

Step 5: Take one code C_1 out of all M codes such that maximum non-zero shift auto-correlation of code C_1 is less than or equal to auto-correlation constraint λ_a of desired sets as input in step one. Calculate cross-correlation of pair of codes formed with other M-1 codes such that in each pair one code is C_1 . Out of M-1 pair of codes only M_1 codes pairing with C_1 are selected which have cross correlation less than or equal to cross correlation constraint λ_c .

Step 6: Repeat step 5 for code C_2 out of all M_1 codes. Get M_2 codes pairing with C_2 out of $(M_1 - 1)$ pair of codes. The step 6 is repeated till the C_{z-1} . Where Z is defined and given as maximum number of codes in the code set formed for given code parameters $(L \times N, w, \lambda_a, \lambda_c)$ such that $(C_1, C_2, \dots, C_{z-1})$ have cross-correlation constraint less than or equal to λ_c . There are total M_{z-1} code which have their cross-correlation value with code C_{z-1} less than or equal to λ_c . Each of these M_{z-1} codes may be treated as code C_z so that there are M_{z-1} set of codes may be formed as



$(C_1, C_2, \dots, C_{z-1}, C_z)$

Step 7: The step 6 may be repeated for all possible other codes C_1 to C_{z-1} which are not employed in last steps to get different set of codes following correlation properties.

5.4.4 Computational Complexity of Algorithm – two

The computational complexity of step 1 to step 7 is remain same as algorithm – one but the value of r is changed for given auto-correlation λ_a and cross-correlation constraint λ_c .

$$\lambda = \max(\lambda_a, \lambda_c)$$

$$r = \frac{(LN-1)(LN-2)\dots(LN-\lambda)}{w(w-1)(w-2)\dots(w-\lambda)} \approx \left(\frac{LN}{w}\right)^\lambda$$

The overall computational complexity of the proposed algorithm - two is of the higher order of $O(r^3)$ which is equivalent $O\left(\left(\frac{LN}{w}\right)^{3\lambda}\right)$ which may be polynomial type for $w \ll LN$ but less complex than algorithm - one.

5.5 Comparison with Ideal Scheme

Both the algorithms proposed here for generation of two dimensional unipolar (optical) orthogonal codes can be compared with an ideal scheme supposed already in chapter – 4 for comparison purpose. The comparison with ideal scheme provide a level of closeness with ideal scheme for generation of two dimensional unipolar orthogonal codes. In following table these schemes are compared with an ideal scheme.

OOC(2 D)	Matrix dimensions (LxN)	Weight 'w'	Auto-correlation constraint λ_a	Cross-correlation constraint λ_c	Cardinality of code-set	No. of code sets	Computational complexity	Other comments
Algorithm - One	$L > 0, N > 0$	$0 < w < L$ N	1 to w-1	1 to w-1	Z	Some random sets	$O\left(\left(\frac{LN}{w}\right)^{3w}\right)$	
Algorithm - Two	$L > 0, N > 0$	$0 < w < L$ N	1 to w-1	1 to w-1	Z	Some random sets	$O\left(\left(\frac{LN}{w}\right)^{3\lambda}\right)$	
Ideal Scheme	$L > 0, N > 0$	$0 < w < L$ N	1 to w-1	1 to w-1	Z	All possible	$\ll O\left((LN)^w\right)$	This scheme is not in existence

Table 5.1: Comparison of proposed algorithms with ideal scheme for generating 2-D U(O)OCs



5.6 Conclusion

The proposed algorithms are able to generate some random sets of two dimensional unipolar orthogonal codes for given code matrix dimensions $L \times N$, code weight 'w' and auto-correlation constraints lying from 1 to w-1. The ideal scheme should generate all possible sets but with very low computational complexity. These proposed schemes are very close to ideal scheme but lack only in the case of computational complexity which may be improved in future.

These two dimensional or matrix orthogonal codes have lower temporal length than one dimensional orthogonal codes with same cardinality. So that the two dimensional codes require wider optical pulse width in comparison to one dimensional codes. Some more comparisons of two dimensional with one dimensional orthogonal codes are given in next chapter along-with future scope.



CHAPTER 6

6. CONCLUSION

In the conclusion of the thesis, the designed one dimensional and then two dimensional orthogonal codes are compared for code parameters, cardinality of the code sets, and computational complexity of the proposed algorithms. The advantages and disadvantages of these generated codes are described as compare to already proposed schemes generating one dimensional and two dimensional optical orthogonal codes and code sets. The future scope of the work proposed is described as follows.

6.1 Advantages and disadvantages of U(O)OCs (1-D & 2-D)

Basically either one dimensional or two dimensional unipolar (optical) orthogonal codes are employed for the purpose of signature sequences to incoherent optical CDMA systems. The one dimensional unipolar (optical) orthogonal codes have been designed here in multiple sets for general values of code parameters. The multiple sets of these codes provide flexibility in selection of code set which ultimately increases the inherent security of the system [152, 154]. These multiple sets may also be utilized for increasing spectral efficiency of the system [137, 143]. The general value of code parameters give us freedom to design the codes as per system's requirement not to design the system as per codes. These codes are generated in sets with maximum cardinality with real upper bounds due to clique search algorithms. The disadvantage of the methods proposed here to design one dimensional unipolar orthogonal codes is mainly computational complexity of the search algorithms. The computational complexity of the algorithms make the upper limits on code parameters so that these algorithms are unable to design the codes for higher values of code parameters (n, w).

The two dimensional unipolar orthogonal codes in their multiple sets are also designed here for general code parameters. The general values of code parameters again provide freedom to design the codes as per system's requirement as well as the multiple sets of codes provide flexibility for selection of codes to increase inherent security and spectral efficiency [133]. These two dimensional codes are designed in sets with maximum cardinality because of search algorithms to find the codes. Again here the high computational complexity of search algorithms is main drawback to be reduced in future works.

These one dimensional as well as two dimensional unipolar orthogonal codes are represented uniquely named difference of positions representation which remain same on every temporal shift of the code. The proposed simple method of calculations of auto-correlation and cross-correlation constraints unig DoPR of the codes reduces computational complexity of orthogonal codes designed in multiple sets.

The multiple access interference or probability of error is directly proportional to correlation constraints (λ_a, λ_c). The multiple access interference can be minimized by setting



the value of $(\lambda_a=1, \lambda_c=1)$ but compromise with lower cardinality or maximum number of codes generated in the set. While with increasing values of correlation constraints $(1 < \lambda_a < w, 1 < \lambda_c < w)$, the cardinality of the system can be increased but with the cost of orthogonality which increases the MAI or probability of error or BER.

6.2 Comparisons of U(O)OCs (1-D & 2-D)

The temporal length of codes directly related to required bandwidth of optical channel or spectral efficiency of channel. The temporal length of two dimensional codes is always less than one dimensional codes for same other code parameters and same cardinality of code sets. Hence spectral efficiency for two dimensional codes is much better than one dimensional codes [133]. The two dimensional unipolar orthogonal codes also provide better inherent security in comparison to one dimensional orthogonal codes because of multi-wavelengths used in a two dimensional or matrix code. Here the two dimensional orthogonal codes with matrix dimension $(L \times N)$ are designed through one dimensional orthogonal codes with code length $n = LN$ and same weight w . It generate the two dimensional codes with lower temporal length than one dimensional orthogonal codes and almost equal computational complexity of the algorithms as given below. The search methods used in algorithm one and two to find one dimensional unipolar orthogonal codes remain same as in algorithm one and two respectively to find two dimensional unipolar orthogonal codes. The conversion of one dimensional to two dimensional is with lower computational complexity so that two dimensional orthogonal codes designing is not more complex than one dimensional codes.

In Appendix I, the result of algorithm one for design of one dimensional unipolar orthogonal code for given code parameters $(n=31, w=3, \lambda_a=1, \lambda_c=1)$. It gives 13 sets of one dimensional unipolar orthogonal codes. Each code set contains 5 codes which is upper bound of the set as per Johnson bounds. In Appendix II, the result of algorithm two for design of one dimensional unipolar orthogonal code for given code parameters $(n=131, w=3, \lambda_a=1, \lambda_c=1)$. The algorithm two uses clique search method than algorithm one which uses direct search method to find multiple sets of one dimensional unipolar orthogonal codes.

In Appendix III, the results of algorithm one for design of two dimensional orthogonal codes for given codes parameters $(L=4, N=3, w=3, \lambda_a=2, \lambda_c=2)$ while Appendix IV have result of algorithm two for design of two dimensional unipolar orthogonal codes for given code parameters $(L=4, N=10, w=3, \lambda_a=2, \lambda_c=2)$ using clique search method.

6.3 Future Scope of the Work

In thesis the author has developed some search algorithms to design one dimensional as well as two dimensional unipolar (optical) orthogonal codes and their comparisons. These codes are generally employed in incoherent optical CDMA systems. During developing the algorithms the author has found many advantages of these codes over already proposed codes in literature and single disadvantage which is high computational complexity of



the search algorithms. In future this drawback may be considered as challenges to take advantages of codes already mentioned. These codes may be utilized in optical LAN and other system where unipolar orthogonal codes are required. One may develop three or multi dimensional unipolar orthogonal codes to increase inherent security and spectral efficiency of the system. One may also develop multi-length and multi-weight one dimensional codes as well as multi matrix dimensions and muti weight two dimensional or multi-dimensional unipolar orthogonal codes to fully access optical bandwidth by multiple users.

U(O)OC	Code length (n) or Matrix dimensions (LxN)	Weight 'w'	Auto-correlation constraint λ_a	Cross-correlation constraint λ_c	Cardinality of code-set	No. of code sets	Computational complexity	Other comments
1-D Algorithm - One	$n > 0$	$0 < w < n$	1 to w-1	1 to w-1	Z	Some random sets	$O\left(\left(\frac{n}{w}\right)^{3w}\right)$	co-prime (w,n)
1-D Algorithm - Two	$n > 0$	$0 < w < n$	1 to w-1	1 to w-1	Z	Some random sets	$O\left(\left(\frac{n}{w}\right)^{3\lambda}\right)$	co-prime (w,n)
2-D Algorithm - One	$L > 0, N > 0$	$0 < w < L, N$	1 to w-1	1 to w-1	Z	Some random sets	$O\left(\left(\frac{LN}{w}\right)^{3w}\right)$	
2-D Algorithm - Two	$L > 0, N > 0$	$0 < w < L, N$	1 to w-1	1 to w-1	Z	Some random sets	$O\left(\left(\frac{LN}{w}\right)^{3\lambda}\right)$	

Table 6.1: Comparison of proposed algorithms for generating 1-D and 2-D U(O)OCs



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APPENDIX I

Results of Algorithm one designing one dimensional unipolar (optical) orthogonal codes for desired code parameters

Code length ('n'), code weight ('w'), total number of codes generated in difference of positions representation with auto-correlation constraint equal to one ('dops'), total number of codes possible as per Johnson's bound ('jb'), auto-correlation constraint for the desired set of codes ('la'), cross-correlation constraint for the desired set of codes ('lc'), as per Johnson bound maximum size of set of codes = $(n-1)/w(w-1)$ = ('gsused'), the designed set of codes with maximum size ('groups').

n= 31

w= 3

dops= 65

jb= 145

la= 1

lc= 1

gsused= 5

groups= 13

Code

S.No. Difference of Positions of codes with 'la'= 1

1	1	2	28
2	1	3	27
3	1	4	26
4	1	5	25
5	1	6	24
6	1	7	23
7	1	8	22
8	1	9	21
9	1	10	20
10	1	11	19



11	1	12	18
12	1	13	17
13	1	14	16
14	2	3	26
15	2	4	25
16	2	5	24
17	2	6	23
18	2	7	22
19	2	8	21
20	2	9	20
21	2	10	19
22	2	11	18
23	2	12	17
24	2	13	16
25	2	14	15
26	3	4	24
27	3	5	23
28	3	6	22
29	3	7	21
30	3	8	20
31	3	9	19
32	3	10	18
33	3	11	17
34	3	12	16
35	3	13	15
36	4	5	22



37	4	6	21
38	4	7	20
39	4	8	19
40	4	9	18
41	4	10	17
42	4	11	16
43	4	12	15
44	4	13	14
45	5	6	20
46	5	7	19
47	5	8	18
48	5	9	17
49	5	10	16
50	5	11	15
51	5	12	14
52	6	7	18
53	6	8	17
54	6	9	16
55	6	10	15
56	6	11	14
57	6	12	13
58	7	8	16
59	7	9	15
60	7	10	14
61	7	11	13
62	8	9	14



63	8	10	13
64	8	11	12
65	9	10	12

**Generated set of codes with their code serial numbers for given code parameters
(n=31, w=3, la=1, lc=1)**

1	38	49	57	62
1	38	51	54	63
1	39	48	55	61
1	44	45	58	65
2	19	51	54	61
2	21	47	56	59
2	21	50	52	62
2	22	46	55	62
2	23	45	59	63
3	18	34	56	63
3	19	33	57	59
3	24	28	60	64
3	25	30	52	65

Any one of these groups can be selected for code assignment to users of optical cdma system.

For example set one is selected with code number [1, 38, 49, 57, 62]

Code No. 1 = DoPR (1, 2, 28) = WPR (0,1,3) = BS [11010000000000000000000000000000]

Code No. 38 = DoPR (4,7,20) = WPR (0,4,11) = BS [10001000000100000000000000000000]

Code No. 49 = DoPR (5,10,16) = WPR (0,5,15) = BS [10000100000000010000000000000000]

Code No. 57 = DoPR (6,12,13) = WPR (0,6,18) = BS [10000010000000000001000000000000]

Code No. 62 = DoPR (8,9,14) = WPR (0,8,17) = BS [10000000100000000100000000000000]



APPENDIX II

Results of Algorithm two designing one dimensional unipolar (optical)

orthogonal codes for desired code parameters

code length ('n'), code weight ('w'), total number of codes generated in difference of positions representation with auto-correlation constraint equal to one ('dops'), total number of codes possible as per Johnson's bound ('jb'), auto-correlation constraint for the desired set of codes ('la'), cross-correlation constraint for the desired set of codes ('lc'), as per Johnson bound maximum size of set of codes = $(n-1)/w(w-1)$ = ('gsused'), the designed set of codes with maximum size ('groups').

n= 131

w= 3

dops= 1365

jb= 2795

la= 1

lc= 1

gsused= 19

groups= 10

Code
S.No. Difference of Positions of codes with 'la'= 1

1	1	2	128
2	1	3	127
3	1	4	126
4	1	5	125
5	1	6	124



1362	41	43	47
1363	41	44	46
1364	42	43	46
1365	42	44	45

**Generated set of codes with their code serial numbers for given code parameters
(n=131, w=3, la=1, lc=1)**

1	186	302	413	566	706	792	910	981	1076
1132	1183	1255	1275	1306	1320	1332	1335	1345	
1	186	302	413	566	706	792	910	981	1076
1132	1183	1255	1276	1308	1318	1325	1341	1345	
1	186	302	413	566	706	792	910	981	1076
1132	1183	1255	1278	1303	1320	1329	1335	1348	
1	186	302	413	566	706	792	910	981	1076
1132	1183	1255	1278	1307	1316	1325	1339	1348	
1	186	302	413	566	706	792	910	981	1076
1132	1183	1255	1279	1303	1318	1330	1335	1349	
1	186	302	413	566	706	792	910	981	1076
1132	1183	1255	1279	1306	1316	1325	1339	1349	
1	186	302	413	566	706	792	910	981	1076
1132	1183	1258	1273	1308	1319	1325	1341	1344	
1	186	302	413	566	706	792	910	981	1076
1132	1183	1258	1278	1301	1315	1330	1341	1345	
1	186	302	413	566	706	792	910	981	1076
1132	1183	1258	1278	1303	1319	1323	1341	1356	
1	186	302	413	566	706	792	910	981	1076
1132	1183	1258	1279	1301	1315	1330	1337	1349	



y one of these groups can be selected for code assignment to users of optical cdma system.

For example set one is selected with code number

[1, 186, 302, 413, 566, 706, 792, 910, 981, 1076, 1132, 1183, 1255, 1275, 1306, 1320, 1332, 1335, 1345]

[illegible]

Similarly

Code No. 186 = DoPR (4,5,122) = WPR (0,4,9) = BS [can be generated similar to code no. 1]

Code No. 302 = DoPR (6,7,118) = WPR (0,6,13)= BS [can be generated similar to code no. 1]

Code No. 413 = DoPR (8,10,113) = WPR (0,8,18) = BS [can be generated similar to code no. 1]

Code No. 566 = DoPR(11,12,108) = WPR (0,11,23) = BS [can be generated similar to code no. 1]

Code No. 706 = DoPR (14,15,102) = WPR (0,14,29)= BS [can be generated similar to code no. 1]

Code No. 792 = DoPR (16,17,98) = WPR (0,16,33)= BS [can be generated similar to code no. 1]

Code No. 910 = DoPR (19,20,92) = WPR (0,19,39)= BS [can be generated similar to code no. 1]

Code No. 981 = DoPR (21,22,88) = WPR (0,21,43)= BS [can be generated similar to code no. 1]

Code No. 1076 = DoPR (24,25,82) = WPR (0,24,49)= BS [can be generated similar to code no. 1]

Code No. 1132 = DoPR (26,27,78) = WPR (0,26,53)= BS [can be generated similar to code no. 1]

Code No. 1183 = DoPR (28,30,73) = WPR (0,28,58)= BS [can be generated similar to code no. 1]

Code No. 1255 = DoPR (31,41,59) = WPR (0,31,72)= BS [can be generated similar to code no. 1]

Code No. 1275 = DoPR (32,44,55) = WPR (0,32,76)= BS [can be generated similar to code no. 1]

Code No. 1306 = DoPR (34,45,52) = WPR (0,34,79)= BS [can be generated similar to code no. 1]

Code No. 1320 = DoPR (35,46,50) = WPR (0,35,81)= BS [can be generated similar to code no. 1]

Code No. 1332 = DoPR (36,37,48) = WPR (0,36,73)= BS [can be generated similar to code no. 1]

Code No. 1335 = DoPR (37,40,54) = WPR (0,37,77)= BS [can be generated similar to code no. 1]

Code No. 1345 = DoPR (38,42,51) = WPR (0,38,80)= BS [can be generated similar to code no. 1]



APPENDIX III

Results of algorithm one for designing of two dimensional unipolar orthogonal codes for code parameters $(L = 4, N = 3, w = 3, \lambda_a = 2, \lambda_c = 2)$ through one dimensional orthogonal codes $(n = LN = 12, w = 3)$

Code S.No.	Difference of Positions of 1D UOC		
1	1	1	10
2	1	2	9
3	1	3	8
4	1	4	7
5	1	5	6
6	2	1	9
7	2	2	8
8	2	3	7
9	2	4	6
10	2	5	5
11	3	1	8
12	3	2	7
13	3	3	6
14	3	4	5
15	4	1	7
16	4	2	6
17	4	3	5
18	5	1	6



Serial no 1

one_dimensional_orthogonal_code = DoPR [1 1 10] = WPR [0,1,2]

one_dimensional_binary_code = 1 1 1 0 0 0 0 0 0 0 0 0

two_dimensional_orthogonal_code = WPR [1'0, 2'0, 3'0] = DoPR[1'0, 2'0, 3'0]

two_dimensional_binary_code =

1 0 0

1 0 0

1 0 0

0 0 0

Serial no 2

one_dimensional_orthogonal_code = DoPR [1 2 9] = WPR [0,1,3]

one_dimensional_binary_code = 1 1 0 1 0 0 0 0 0 0 0 0

two_dimensional_orthogonal_code = WPR [1'0, 2'0, 4'0] = DoPR[1'0, 2'0, 4'0]

two_dimensional_binary_code =

1 0 0

1 0 0

0 0 0

1 0 0

Serial no 3

one_dimensional_orthogonal_code = DoPR [1 3 8] = WPR [0,1,4]

one_dimensional_binary_code = 1 1 0 0 1 0 0 0 0 0 0 0

two_dimensional_orthogonal_code = WPR [1'0, 2'0, 1'1] = DoPR[1'0, 2'1, 1'2]

two_dimensional_binary_code =

1 1 0



1 0 0
0 0 0
0 0 0

Serial no 4

one_dimensional_orthogonal_code = DoPR [1 4 7] = WPR [0,1,5]

one_dimensional_binary_code = 1 1 0 0 0 1 0 0 0 0 0 0

two_dimensional_orthogonal_code = WPR [1'0, 2'0, 2'1] = DoPR[1'0, 2'1, 2'2]

two_dimensional_binary_code =

1 0 0
1 1 0
0 0 0
0 0 0

Serial no 5

one_dimensional_orthogonal_code = DoPR [1 5 6] = WPR [0,1,6]

one_dimensional_binary_code = 1 1 0 0 0 0 1 0 0 0 0 0

two_dimensional_orthogonal_code = WPR [1'0, 2'0, 3'1] = DoPR[1'0, 2'1, 3'2]

two_dimensional_binary_code =

1 0 0
1 0 0
0 1 0
0 0 0

Serial no 6

one_dimensional_orthogonal_code = DoPR [2 1 9] = WPR [0,2,3]

one_dimensional_binary_code = 1 0 1 1 0 0 0 0 0 0 0 0

two_dimensional_orthogonal_code = WPR [1'0, 3'0, 4'0] = DoPR[1'0, 3'0, 4'0]

two_dimensional_binary_code =



1 0 0

0 0 0

1 0 0

1 0 0

Serial no 7

one_dimensional_orthogonal_code = DoPR [2 2 8] = WPR [0,2,4]

one_dimensional_binary_code = 1 0 1 0 1 0 0 0 0 0 0 0

two_dimensional_orthogonal_code = WPR [1'0, 3'0, 1'1] = DoPR[1'0, 3'1, 1'2]

two_dimensional_binary_code =

1 1 0

0 0 0

1 0 0

0 0 0

Serial no 8

one_dimensional_orthogonal_code = DoPR [2 3 7] = WPR [0,2,5]

one_dimensional_binary_code = 1 0 1 0 0 1 0 0 0 0 0 0

two_dimensional_orthogonal_code = WPR [1'0, 3'0, 2'1] = DoPR[1'0, 3'1, 2'2]

two_dimensional_binary_code =

1 0 0

0 1 0

1 0 0

0 0 0

Serial no 9

one_dimensional_orthogonal_code = DoPR [2 4 6] = WPR [0,2,6]

one_dimensional_binary_code = 1 0 1 0 0 0 1 0 0 0 0 0

two_dimensional_orthogonal_code = WPR [1'0, 3'0, 3'1] = DoPR[1'0, 3'1, 3'2]



two_dimensional_binary_code =

1 0 0
0 0 0
1 1 0
0 0 0

Serial no 10

one_dimensional_orthogonal_code = DoPR [2 5 5] = WPR [0,2,7]

one_dimensional_binary_code = 1 0 1 0 0 0 0 1 0 0 0 0

two_dimensional_orthogonal_code = WPR [1'0, 3'0, 4'1] = DoPR[1'0, 3'1, 4'2]

two_dimensional_binary_code =

1 0 0
0 0 0
1 0 0
0 1 0

Serial no 11

one_dimensional_orthogonal_code = DoPR [3 1 8] = WPR [0,3,4]

one_dimensional_binary_code = 1 0 0 1 1 0 0 0 0 0 0 0

two_dimensional_orthogonal_code = WPR [1'0, 4'0, 1'1] = DoPR[1'0, 4'1, 1'2]

two_dimensional_binary_code =

1 1 0
0 0 0
0 0 0
1 0 0

Serial no 12

one_dimensional_orthogonal_code = DoPR [3 2 7] = WPR [0,3,5]

one_dimensional_binary_code = 1 0 0 1 0 1 0 0 0 0 0 0



two_dimensional_orthogonal_code = WPR [1'0, 4'0, 2'1] = DoPR[1'0, 4'1, 2'2]

two_dimensional_binary_code =

1 0 0

0 1 0

0 0 0

1 0 0

Serial no 13

one_dimensional_orthogonal_code = DoPR [3 3 6] = WPR [0,3,6]

one_dimensional_binary_code = 1 0 0 1 0 0 1 0 0 0 0 0

two_dimensional_orthogonal_code = WPR [1'0, 4'0, 3'1] = DoPR[1'0, 4'1, 3'2]

two_dimensional_binary_code =

1 0 0

0 0 0

0 1 0

1 0 0

Serial no 14

one_dimensional_orthogonal_code = DoPR [3 4 5] = WPR [0,3,7]

one_dimensional_binary_code = 1 0 0 1 0 0 0 1 0 0 0 0

two_dimensional_orthogonal_code = WPR [1'0, 4'0, 4'1] = DoPR[1'0, 4'1, 4'2]

two_dimensional_binary_code =

1 0 0

0 0 0

0 0 0

1 1 0

Serial no 15

one_dimensional_orthogonal_code = DoPR [4 1 7] = WPR [0,4,5]



one_dimensional_binary_code = 1 0 0 0 1 1 0 0 0 0 0 0

two_dimensional_orthogonal_code = WPR [1'0, 1'1, 2'1] = DoPR[1'1, 1'0, 2'2]

two_dimensional_binary_code =

1 1 0

0 1 0

0 0 0

0 0 0

Serial no 16

one_dimensional_orthogonal_code = DoPR [4 2 6] = WPR [0,4,6]

one_dimensional_binary_code = 1 0 0 0 1 0 1 0 0 0 0 0

two_dimensional_orthogonal_code = WPR [1'0, 1'1, 3'1] = DoPR[1'1, 1'0, 3'2]

two_dimensional_binary_code =

1 1 0

0 0 0

0 1 0

0 0 0

Serial no 17

one_dimensional_orthogonal_code = DoPR [4 3 5] = WPR [0,4,7]

one_dimensional_binary_code = 1 0 0 0 1 0 0 1 0 0 0 0

two_dimensional_orthogonal_code = WPR [1'0, 1'1, 4'1] = DoPR[1'1, 1'0, 4'2]

two_dimensional_binary_code =

1 1 0

0 0 0

0 0 0

0 1 0



Serial no 18

one_dimensional_orthogonal_code = DoPR [5 1 6] = WPR [0,5,6]

one_dimensional_binary_code = 1 0 0 0 0 1 1 0 0 0 0 0

two_dimensional_orthogonal_code = WPR [1'0, 2'1, 3'1] = DoPR[1'1, 2'0, 3'2]

two_dimensional_binary_code =

1 0 0

0 1 0

0 1 0

0 0 0



APPENDIX IV

Results of algorithm two for designing of two dimensional unipolar orthogonal codes for code parameters $(L = 10, N = 4, w = 5, \lambda_a = 4, \lambda_c = 4)$ through one dimensional orthogonal codes $(n = LN = 40, w = 5)$

Code S.No.	Difference of Positions of 1D UOC				
1	1	1	1	1	36
2	1	1	1	2	35
3	1	1	1	3	34
4	1	1	1	4	33
5	1	1	1	5	32

16447	17	2	1	1	19
16448	17	2	1	2	18
16449	17	2	2	1	18
16450	17	3	1	1	18
16451	18	1	1	1	19

Serial no 1

one_dimensional_orthogonal_code = DoPR [1 1 1 1 36] = WPR [0,1,2,3,4]

one_dimensional_binary_code =

Columns 1 through 17

1 1 1 1 1 0 0 0 0 0 0 0 0 0 0 0

Columns 18 through 34



0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0

Columns 35 through 40

0 0 0 0 0 0

2_dimensional_orthogonal_code = WPR[1'0, 2'0, 3'0, 4'0, 1'1] = DoPR[1'0, 2'0, 3'0, 4'1, 1'9]

two_dimensional_binary_code =

1 1 0 0 0 0 0 0 0 0

1 0 0 0 0 0 0 0 0 0

1 0 0 0 0 0 0 0 0 0

1 0 0 0 0 0 0 0 0 0

Serial no 2

one_dimensional_orthogonal_code = DoPR [1 1 1 2 35] = WPR [0,1,2,3,5]

one_dimensional_binary_code =

Columns 1 through 17

1 1 1 1 0 1 0 0 0 0 0 0 0 0 0 0 0

Columns 18 through 34

0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0

Columns 35 through 40

0 0 0 0 0 0

2_dimensional_orthogonal_code = WPR[1'0, 2'0, 3'0, 4'0, 2'1] = DoPR[1'0, 2'0, 3'0, 4'1, 2'9]

two_dimensional_binary_code =

1 0 0 0 0 0 0 0 0 0

1 1 0 0 0 0 0 0 0 0

1 0 0 0 0 0 0 0 0 0

1 0 0 0 0 0 0 0 0 0



Serial no 3

one_dimensional_orthogonal_code = DoPR [1 1 1 3 34] = WPR [0,1,2,3,6]

one_dimensional_binary_code =

Columns 1 through 17

1 1 1 1 0 0 1 0 0 0 0 0 0 0 0 0 0

Columns 18 through 34

0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0

Columns 35 through 40

0 0 0 0 0 0

2_dimensional_orthogonal_code = WPR[1'0, 2'0, 3'0, 4'0, 3'1] = DoPR[1'0, 2'0, 3'0, 4'1, 3'9]

two_dimensional_binary_code =

1 0 0 0 0 0 0 0 0 0

1 0 0 0 0 0 0 0 0 0

1 1 0 0 0 0 0 0 0 0

1 0 0 0 0 0 0 0 0 0

Serial no 16449

one_dimensional_orthogonal_code = DoPR [17 2 2 1 18] = WPR [0,17,19,21,22]

one_dimensional_binary_code=

Columns 1 through 17

1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0

Columns 18 through 34

1 0 1 0 1 1 0 0 0 0 0 0 0 0 0 0 0

Columns 35 through 40



0 0 0 0 0 0

2_dimensional_orthogonal_code = WPR[1'0, 2'0, 4'0, 2'0, 3'1] = DoPR[1'4, 2'0, 4'1, 2'0, 3'5]

two_dimensional_binary_code =

1 0 0 0 0 0 0 0 0 0
 0 0 0 0 1 1 0 0 0 0
 0 0 0 0 0 1 0 0 0 0
 0 0 0 0 1 0 0 0 0 0

Serial no 16450

one_dimensional_orthogonal_code = DoPR [17 3 1 1 18] = WPR [0,17,20,21,22]

one_dimensional_binary_code=

Columns 1 through 17

1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0

Columns 18 through 34

1 0 0 1 1 1 0 0 0 0 0 0 0 0 0 0 0

Columns 35 through 40

0 0 0 0 0 0

2_dimensional_orthogonal_code = WPR[1'0, 2'4, 1'5, 2'5, 3'5] = DoPR[1'4, 2'1, 1'0, 2'0, 3'5]

two_dimensional_binary_code =

1 0 0 0 0 1 0 0 0 0
 0 0 0 0 1 1 0 0 0 0
 0 0 0 0 0 1 0 0 0 0
 0 0 0 0 0 0 0 0 0 0

Serial no 16451

one_dimensional_orthogonal_code = DoPR [18 1 1 1 19] = WPR [0,18,19,20,21]

one_dimensional_binary_code=

Columns 1 through 17



1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0

Columns 18 through 34

0 1 1 1 1 0 0 0 0 0 0 0 0 0 0 0 0

Columns 35 through 40

0 0 0 0 0 0

2_dimensional_orthogonal_code = WPR[1'0, 3'4, 4'4, 1'5, 2'5] = DoPR[1'4, 3'0, 4'1, 1'0, 2'5]

two_dimensional_binary_code =

1 0 0 0 0 1 0 0 0 0

0 0 0 0 0 1 0 0 0 0

0 0 0 0 1 0 0 0 0 0

0 0 0 0 1 0 0 0 0 0

Total codes formed = 16451

These generated codes can be assigned to different multiple users accessing the common optical channel using CDMA scheme in asynchronous manner.



PUBLICATIONS FROM THIS RESEARCH WORK

- [1] R. C. S. Chauhan, R. Asthana, Y. N. Singh, “A General Algorithm to Design Sets of All Possible One Dimensional Unipolar orthogonal codes of Same Code Length and Weight,” 2010 IEEE International Conference on Computational Intelligence and Computing Research (ICCIC-2010), Coimbatore, India, *IEEE conference proceedings*, 978-1-4244-5966-7/10, 28-29 December 2010, pp. 7-13.
- [2] R. C. S. Chauhan, R. Asthana, Y. N. Singh, “Unipolar Orthogonal Codes: Design, Analysis and Applications” International Conference on High Performance Computing (HiPC-2010), *Student Research Symposium*, 19-22 December 2010, Goa, India.
- [3] R. C. S. Chauhan, R. Asthana, “Representation and calculation of correlation constraints of one dimensional unipolar orthogonal codes (1-D UOC),” IEEE International Conference CSNT-2011, Jammu, India on 3 – 5 June 2011. *IEEE conference proceedings*, 978-1-4577-0543-4, 3 – 5 June 2011, pp. 483-489.
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- [5] R. C. S. Chauhan, Y. N. Singh, R. Asthana, “Design of Two Dimensional Unipolar (Optical) Orthogonal Codes Through One Dimensional Unipolar (Optical) Orthogonal Codes,” *Journal of Computing Technologies*, Vol 2, Issue 9, September 2013, pp. 20-24.
- [6] R. C. S. Chauhan, Y. N. Singh, R. Asthana, “Design of Minimum Correlated, Maximal Clique Sets of One Dimensional Unipolar (Optical) Orthogonal codes” *arXiv preprint arxiv: 1309.0193*, 2013.
- [7] R. C. S. Chauhan, Y. N. Singh, R. Asthana, “Design of Minimum Correlated, Maximal Clique Sets of Two Dimensional Unipolar (Optical) Orthogonal codes” Under Review.



Biography of Research Scholar



Ram Chandra Singh Chauhan s/o Shri K. N. Singh, born on January 15, 1980, at Shivnagar (Ramnagar), Mandhana, Kanpur. He received his B.Tech in Electronics Engineering from Institute of Engineering & Technology, Lucknow, University of Lucknow, (U.P.), India in 2001. He received his M.Tech in Digital System (Electronics Engineering) from Motilal Nehru National Institute of Technology, Allahabad, (U.P.), India, in 2003. He worked as a Lecturer in Electronics & Communication Engineering department of University Institute of Engineering & Technology, Kanpur from January 2004 to July 2007. He received a Teacher Fellowship by Uttar Pradesh Technical University, Lucknow (U.P.) in July 2007 along-with pursuing his Ph.D. research work. He had worked as Teacher-Fellow in Electronics Engineering department of H.B.T.I., Kanpur since July 2007 to July 2012. He is presently working as Assistant Professor in Electronics & Communication Engineering department of Pranveer Singh Institute of Technology, Kanpur, India since July 2012 onwards. His interests are in Multiple Access Schemes for Optical Channel, Orthogonal Coding Theory, Digital Signal Processing and Information Theory and Coding. He has published ten research papers in different international journals and conference proceedings including the publications from present PhD research work.

